●●●○○ Level 3 · Mathematics M3.4

Ratio and proportion

Compare amounts with ratios, share in a ratio, and scale recipes and maps.

Write and simplify ratios, link them to fractions, and share an amount in a ratio. Then find the amount for one, tell direct from inverse proportion, work out speeds and best buys, and use the scale on a map.

YOU ARE READY FOR THIS BOOK IF

You can simplify 12/16 to 3/4.

One step up	M4.2 Percentages, proportion and growth
This book	**M3.4 Ratio and proportion**
One step down	M2.4 Fractions

Contents

M3.4.1 Ratio: comparing parts

M3.4.2 Ratios and fractions

M3.4.3 Sharing in a ratio

M3.4.4 Finding one first

M3.4.5 Direct proportion

M3.4.6 Rates and speed

M3.4.7 Maps and scale drawings

M3.4.8 Inverse proportion

Key words, A to Z

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M3.4.1 CHAPTER 1

Ratio: comparing parts

> **Ravi says there are 3 goats for every 2 sheep. I count 6 goats and 4 sheep. Who is right?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Ravi is wrong. The ratio is 6 : 4, so he cannot say 3 : 2.
- ☐ **B** Both are right. 6 : 4 and 3 : 2 describe the same flock.
- ☐ **C** I am wrong. There are really only 3 goats and 2 sheep.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Negative numbers, factors and powers, M3.1.6. To simplify a ratio you divide by the highest common factor of its two numbers.
- Level 2, Fractions, M2.4.6. Equal ratios work like equal fractions: multiply or divide both numbers by the same number.

BY THE END YOU CAN

- Write the ratio of 6 goats to 4 sheep as 6 : 4
- Simplify a ratio like 6 : 4 to 3 : 2

A **ratio** compares two amounts of the same kind. Kofi has 6 goats and 4 sheep. The ratio of goats to sheep is 6 : 4. We read it as 6 to 4.

The order matters. The ratio of sheep to goats is 4 : 6. It is not the same as 6 : 4.

DRAW IT FIRST

Draw the flock as dots

goats



1 Kofi has 6 goats. Draw a filled dot for each goat.

goats



sheep



2 Kofi has 4 sheep. Draw an empty dot for each sheep.



each group is 3 : 2

3 Put the dots in 2 equal groups. Each group has 3 goats and 2 sheep.

Each group has 3 goats and 2 sheep. So for every 3 goats there are 2 sheep. The ratio 3 : 2 says the same as 6 : 4. We call them **equal ratios**.

To simplify a ratio, divide both numbers by their highest common factor. You met this in the chapter on prime factors. A ratio is in its **simplest form** when only 1 goes into both numbers.

WORKED EXAMPLE

Write 12 : 18 in simplest form.

1. The highest common factor of 12 and 18 is 6.
2. 12 ÷ 6 = 2.
3. 18 ÷ 6 = 3.
4. In simplest form the ratio is 2 : 3.

Answer: 2 : 3

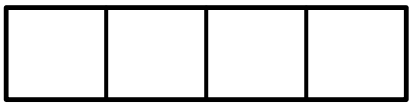
Both amounts must be in the same unit before you simplify. You cannot compare metres with centimetres. Change one so that both match.

WORKED EXAMPLE

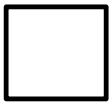
Write the ratio of 2 m to 50 cm in simplest form.

1. Make the units match. 2 m = 200 cm.
2. The ratio is 200 : 50.
3. The highest common factor of 200 and 50 is 50.
4. 200 ÷ 50 = 4 and 50 ÷ 50 = 1.
5. In simplest form the ratio is 4 : 1.

Answer: 4 : 1



rope: 2 m = 200 cm



stick: 50 cm

One part is 50 cm. The rope is 4 parts and the stick is 1 part, so the ratio is 4 : 1.

SEE IT THREE WAYS

With things

Take 3 stones and 2 beans. Make the same mix again and again: 6 stones with 4 beans, then 9 stones with 6 beans.

As a picture

stones	beans
3	2
6	4
9	6
12	8

In symbols

3 : 2 = 6 : 4 = 9 : 6 = 12 : 8. Each row is the first row times 1, 2, 3 and 4.

TRY IT

Make equal ratios with stones and beans

You need: 6 small stones, 4 beans or seeds.

1. Put 6 stones in one row and 4 beans in another row.
2. Split them into piles. Each pile must have the same number of stones and the same number of beans.
3. Find the largest number of piles you can make with nothing left over.
4. Count the stones and beans in one pile.

What you will see: You can make 2 piles. Each pile has 3 stones and 2 beans. The number of piles, 2, is the highest common factor of 6 and 4.

If you cannot try it, this is what happens.



filled: stone, empty: bean

A WRONG IDEA MANY PEOPLE HAVE

~~To make an equal ratio, add the same number to both sides. So 2 : 3 is the same as 3 : 4.~~

Why it feels right: Doing the same thing to both numbers sounds fair. Adding 1 to each looks like it keeps them in step.

The test that breaks it: Take 2 stones and 3 beans. Add one of each to get 3 stones and 4 beans. Now try to split both heaps into equal piles that look like 2 stones with 3 beans. You cannot. Double the first heap instead: 4 stones and 6 beans make 2 piles of 2 and 3.

What is true: Equal ratios come from multiplying or dividing both numbers by the same number. Adding the same number changes the mix.

KEY WORDS

ratio
a way to compare two amounts of the same kind, written like 6 : 4.

equal ratios
ratios that describe the same mix, such as 6 : 4 and 3 : 2.

simplest form
a ratio where only 1 goes into both numbers.

BACK TO YOUR GUESS

The answer is B: Both are right. 6 : 4 and 3 : 2 describe the same flock.

Both are right. Divide 6 and 4 by 2 and you get 3 and 2. The flock has the same mix either way. The ratios 6 : 4 and 3 : 2 are equal.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Each filled dot is a mango. Each empty dot is a lime. What is the ratio of mangoes to limes?



filled: mango, empty: lime

A 3 : 5

B 5 : 3

C 5 : 8

Answer at the back of the book.

2. Which is 15 : 25 in simplest form?

A 3 : 5

B 5 : 3

C 3 : 25

Answer at the back of the book.

3. Which ratio is equal to 4 : 6?

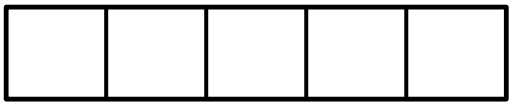
A 6 : 8

B 2 : 3

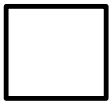
C 4 : 8

Answer at the back of the book.

4. A rope is 3 m long. A stick is 60 cm long. What is the ratio of rope to stick in simplest form?



rope: 3 m = 300 cm



stick: 60 cm

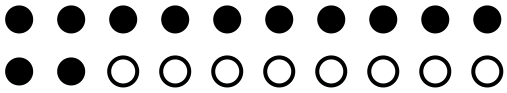
A 3 : 60

B 1 : 20

C 5 : 1

Answer at the back of the book.

5. A bag holds 12 red beads and 8 blue beads. Which ratio of red to blue is in simplest form?



filled: red, empty: blue

A 12 : 8

B 6 : 4

C 3 : 2

Answer at the back of the book.

6. Write 18 : 24 in simplest form.

Answer at the back of the book.

7. A rope is 4 m long. A stick is 50 cm long. Write the ratio of rope to stick in simplest form.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A ratio compares two amounts in order. 6 : 4 is equal to 3 : 2. Divide both numbers by their highest common factor to simplify. Make the units match first.

Draw: Draw 6 filled dots and 4 empty dots. Put them in 2 groups, each with 3 filled and 2 empty. Write 6 : 4 = 3 : 2.

M3.4.2 CHAPTER 2

Ratios and fractions

> **Ana mixes juice and water in the ratio 2 : 3. Is the juice two thirds of the drink?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. The juice is 2 out of 3, so it is two thirds.
- ☐ **B** No. The juice is 2 of the 5 parts, so it is two fifths.
- ☐ **C** No. The juice is 3 of the 5 parts, so it is three fifths.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Fractions, M2.4.3. The top number of a fraction counts the parts you take. The bottom number counts all the equal parts.

BY THE END YOU CAN

- Change a ratio of 2 : 3 into fractions of the whole
- Tell a part to part ratio from a part to whole fraction

In a ratio like 2 : 3, each number counts **parts**. The juice has 2 parts. The water has 3 parts. All the parts are the same size.

A ratio compares one part with another part. We call that **part to part**. A fraction compares one part with the whole. We call that **part to whole**.

DRAW IT FIRST

Find the whole, then the fraction



whole: 5 parts

1 Draw a bar with 5 equal parts. 2 + 3 = 5, so the bar is the whole drink.



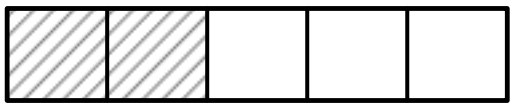
whole: 5 parts

2 Shade 2 parts for the juice.

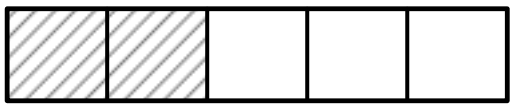


juice 2 **water 3**

3 The juice is 2 of the 5 parts. The water is the other 3.



part to part: 2 : 3



part to whole: 2 of 5

Same bar, two ways to say it. The ratio looks at the plain part too. The fraction looks at the whole bar.

So the juice is $^{2}/_{5}$ of the drink. The water is $^{3}/_{5}$. The bottom number of each fraction is the total of the ratio.

WORKED EXAMPLE

Juice and water are in the ratio 2 : 3. What fraction of the drink is juice?

1. 2 + 3 = 5, so the whole drink is 5 parts.
2. The juice is 2 of the 5 parts.
3. The fraction of juice is $^{2}/_{5}$.

Answer: $^{2}/_{5}$

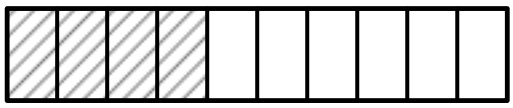
Sometimes the fraction can be made simpler. Simplify it as you did before, by dividing the top and the bottom by the same number.

WORKED EXAMPLE

Red and black seeds are in the ratio 4 : 6. What fraction of the seeds is red? Give it in simplest form.

1. 4 + 6 = 10, so there are 10 parts.
2. Red is 4 of the 10 parts, which is $^{4}/_{10}$.
3. Divide the top and the bottom by 2. This gives $^{2}/_{5}$.

Answer: $^{2}/_{5}$



4 of 10 parts



2 of 5 parts

The two shaded parts end at the same place. $^{4}/_{10}$ and $^{2}/_{5}$ are the same share.

SEE IT THREE WAYS

With things	As a picture	In symbols
Yusuf has 2 red beans and 3 black beans. Put them in one line.		**2 : 3 compares part with part. 2 + 3 = 5. Red is $^{2}/_{5}$ of the beans. Black is $^{3}/_{5}$.**

TRY IT

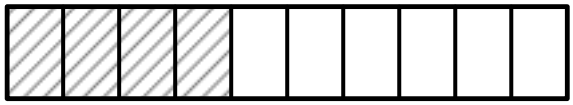
Colour a strip of squares

You need: squared paper, a pencil.

1. Draw a strip of 5 squares. Colour 2 of them. The ratio of coloured to plain is 2 : 3.
2. Count all the squares. The coloured squares are 2 of 5.
3. Now draw a strip of 10 squares. Colour 4 of them.
4. Count the plain squares. Write the ratio of coloured to plain, and the fraction that is coloured.

What you will see: The second strip has 4 coloured and 6 plain squares. The ratio is 4 : 6 and the fraction is 4 out of 10. Both simplify to the first strip: 2 : 3 and 2 out of 5.

If you cannot try it, this is what happens.



shaded 4, plain 6

A WRONG IDEA MANY PEOPLE HAVE

~~The ratio 2 : 3 means the juice is two thirds of the drink.~~

Why it feels right: You can see a 2 and a 3. Two thirds is also written with a 2 and a 3.

The test that breaks it: Draw 2 squares for juice and 3 squares for water in one strip. Count the whole strip. It has 5 squares, not 3. Two thirds of the strip would be more than 3 squares.

What is true: In a ratio, 3 is the water only. The whole drink is 2 + 3 = 5 parts. The juice is two fifths.

KEY WORDS

parts
the equal pieces that the numbers of a ratio count.

part to part
a ratio, which compares one part with another part.

part to whole
a fraction, which compares one part with all the parts.

BACK TO YOUR GUESS

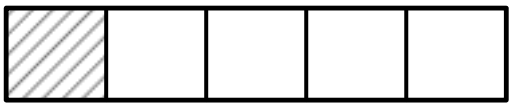
The answer is B: No. The juice is 2 of the 5 parts, so it is two fifths.

The ratio 2 : 3 compares juice with water. The whole drink is juice and water together: 2 + 3 = 5 parts. The juice is 2 of the 5 parts, which is two fifths.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The ratio of goats to sheep is 1 : 4. What fraction of the animals are goats?



goats 1, sheep 4

A $^1/_4$

B $^1/_5$

C $^4/_5$

Answer at the back of the book.

2. The ratio of dogs to cats is 5 : 3. What fraction of the animals are cats?

A $^3/_5$

B $^5/_8$

C $^3/_8$

Answer at the back of the book.

3. $^2/_7$ of a class walk to school. The others come by bus. What is the ratio of walkers to bus riders?

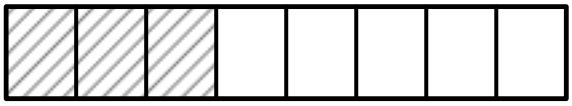
A 2 : 7

B 5 : 2

C 2 : 5

Answer at the back of the book.

4. Look at the bar. What is the ratio of shaded parts to plain parts?



8 equal parts

A 3 : 8

B 3 : 5

C 5 : 3

Answer at the back of the book.

5. Juice and water are in the ratio 2 : 3. Which sentence compares the juice with the whole drink?

   A Juice is $^2/_3$ of the drink.

   B The ratio of juice to water is 2 : 3.

   C Juice is $^2/_5$ of the drink.

   Answer at the back of the book.

6. Red and blue paint are mixed in the ratio 3 : 4. What fraction of the mix is red?

   Answer at the back of the book.

7. $^3/_8$ of a class walk to school. The others come by bus. What is the ratio of walkers to bus riders?

   Answer at the back of the book.

8. **From an earlier chapter**

   Write the ratio 18 : 12 in simplest form.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A ratio compares part with part. A fraction compares a part with the whole. Add the parts to find the whole. In 2 : 3 the whole is 5 parts, and the first part is 2 out of 5.

Draw: Draw a bar of 5 equal squares. Shade 2. Write 2 : 3 under it, and 2 out of 5 above it.

M3.4.3 CHAPTER 3

Sharing in a ratio

> **Amina and Ravi share 35 seeds in the ratio 2 : 5. How many seeds does each one get?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Amina gets 2 seeds and Ravi gets 5 seeds.
- ☐ **B** Amina gets 17 seeds and Ravi gets 18 seeds, almost half each.
- ☐ **C** Amina gets 10 seeds and Ravi gets 25 seeds.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.3. Finding one part is a sharing sum: the total shared into equal parts.

BY THE END YOU CAN

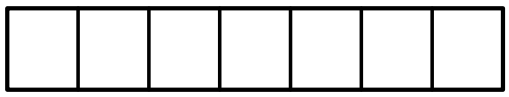
- Share 35 seeds in the ratio 2 : 5
- Find the total when you know the size of one share

To share in a ratio, think of equal parts. The ratio 2 : 5 gives Amina 2 parts and Ravi 5 parts. Every part is the same size.

First add the parts to get the number of equal parts. Then share the **total** by that number to find **one part**. Last, multiply by each share.

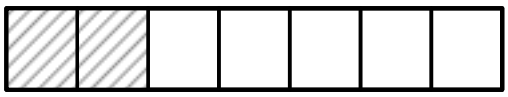
DRAW IT FIRST

Share 35 seeds in the ratio 2 : 5



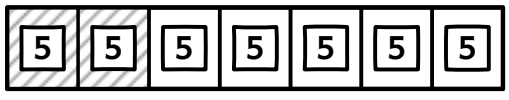
whole: 35 seeds

1 Add the parts: 2 + 5 = 7. Draw a bar of 7 equal parts for the 35 seeds.



Amina 2 **Ravi 5**

2 Amina gets 2 parts and Ravi gets 5 parts.



Amina 2 **Ravi 5**

each part: 5 seeds

3 35 ÷ 7 = 5, so each part is 5 seeds.

WORKED EXAMPLE

Share 35 seeds between Amina and Ravi in the ratio 2 : 5.

1. Add the parts: 2 + 5 = 7.
2. Find one part: 35 ÷ 7 = 5.
3. Amina has 2 × 5 = 10 seeds. Ravi has 5 × 5 = 25 seeds.
4. Check: 10 + 25 = 35.

Answer: Amina gets 10 seeds. Ravi gets 25 seeds.

Sometimes you know one share and need the total. Then work the other way. One share gives you one part. One part gives you the total.

WORKED EXAMPLE

Leila and Tariq share dates in the ratio 3 : 4. Leila gets 12 dates. How many dates are there in all?

1. Leila has 3 parts, and 3 parts are 12 dates.
2. One part is 12 ÷ 3 = 4 dates.
3. There are 3 + 4 = 7 parts in all.
4. The total is 7 × 4 = 28 dates.

Answer: 28 dates



Leila: 3 parts = 12 dates

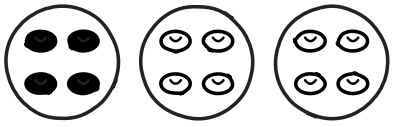
Leila's 3 parts hold 12 dates, so each part holds 4 dates. The whole bar has 7 parts.

SEE IT THREE WAYS

With things

Mei and Diego share 12 beans in the ratio 1 : 2. Deal the beans into 3 equal piles. Mei takes 1 pile and Diego takes 2 piles.

As a picture



Mei Diego Diego

In symbols

1 + 2 = 3 parts. 12 ÷ 3 = 4. Mei gets 1 × 4 = 4 beans. Diego gets 2 × 4 = 8 beans.

TRY IT

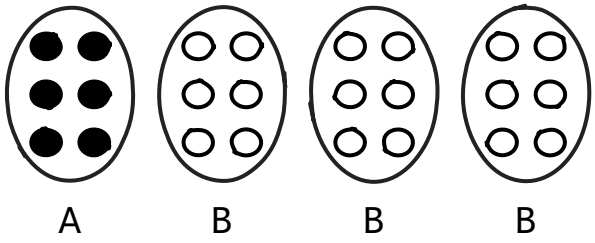
Share 24 stones in the ratio 1 : 3

You need: 24 small stones, or beans.

1. Make a heap of 24 stones.
2. The parts are 1 + 3 = 4. Deal the stones one at a time into 4 piles until none are left.
3. Friend A takes 1 pile. Friend B takes the other 3 piles.
4. Count the stones each friend has.

What you will see: Each pile has 6 stones. Friend A has 6 stones and friend B has 18. 6 + 18 = 24, so nothing is lost.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The ratio 2 : 5 means one person gets 2 seeds and the other gets 5 seeds.~~

Why it feels right: The numbers 2 and 5 are right there, and it is quick to use them as they are.

The test that breaks it: Give 2 seeds to Amina and 5 to Ravi. That uses 2 + 5 = 7 seeds. There were 35, so 28 seeds are still in the heap.

What is true: The numbers in a ratio count equal parts, not seeds. Find how many seeds one part holds. Here one part is 5 seeds.

KEY WORDS

total

the whole amount that is shared.

one part
the amount that each equal part of the ratio holds.

BACK TO YOUR GUESS

The answer is C: Amina gets 10 seeds and Ravi gets 25 seeds.

The ratio 2 : 5 means 2 + 5 = 7 equal parts. 35 seeds in 7 parts is 5 seeds a part. Amina has 2 parts, which is 10 seeds. Ravi has 5 parts, which is 25 seeds.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Share 40 beans between Sipho and Lucía in the ratio 3 : 5. How many beans does each get?

   A 3 and 5

   B 20 and 20

   C 24 and 16

   D 15 and 25

   Answer at the back of the book.

2. Cut a rope 54 cm long into two pieces in the ratio 4 : 5. How long is each piece?

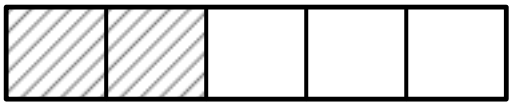
   A 27 cm and 27 cm

   B 36 cm and 45 cm

   C 24 cm and 30 cm

   Answer at the back of the book.

3. Hana and Joseph share stones in the ratio 2 : 3. Hana gets 14 stones. How many stones are there in all?



Hana: 2 parts = 14 stones

A 21 stones

B 28 stones

C 35 stones

Answer at the back of the book.

4. The bar shows 28 sweets shared in the ratio 3 : 4. How many sweets are in the share with 3 parts?



28 sweets in 7 equal parts

A 4 sweets

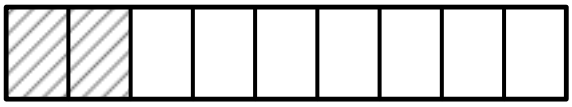
B 16 sweets

C 21 sweets

D 12 sweets

Answer at the back of the book.

5. Red and white paint are mixed in the ratio 2 : 7 to make 36 litres. How much red paint is there?



36 litres in 9 equal parts

A 8 litres

B 4 litres

C 18 litres

D 28 litres

Answer at the back of the book.

6. Share 63 cm of ribbon between Ana and Joseph in the ratio 2 : 7. How long is each piece? Ana has the 2 parts.

   Answer at the back of the book.

7. Ana and Diego share seeds in the ratio 3 : 5. Ana gets 15 seeds. How many seeds are there in all?

   Answer at the back of the book.

8. **From an earlier chapter**

   Black and white beads are in the ratio 3 : 7. What fraction of the beads is black?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Add the parts. Divide the total by the number of parts to find one part. Multiply one part by each share. Check that the shares add up to the total.

Draw: Draw a bar of 7 equal parts for 35 seeds. Shade 2 parts. Write 5 in every part.

Finding one first

> **A recipe makes 4 flat breads with 240 g of flour. How much flour do I need for 10 flat breads?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Add 6 g, because I want 6 more flat breads. So 246 g.
- ☐ **B** Use 240 g for every 4, so 480 g.
- ☐ **C** Find the flour for 1 flat bread, then take 10 of those: 600 g.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.3. Finding the amount for one is a sharing sum.

BY THE END YOU CAN

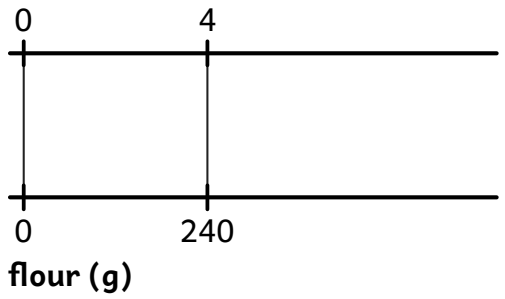
- Find the amount for one, then for any number
- Scale a recipe for 4 flat breads up to 10 flat breads

A recipe says how much of each thing you need for a number of servings. To change the number of servings, find the amount for one first. We call this the **unit amount**. Then multiply.

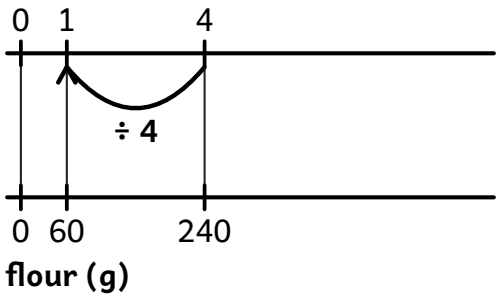
Making more is called to **scale up**. Making less is called to **scale down**. The same two steps work for both.

DRAW IT FIRST

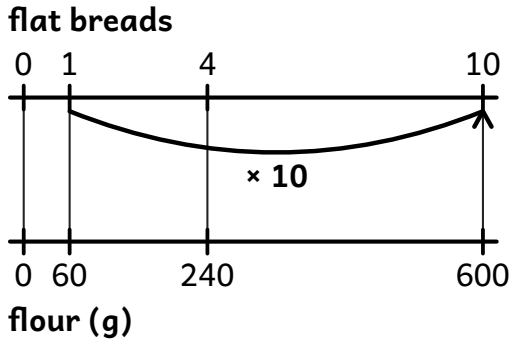
Find one first on *a* double number line



1 4 flat breads need 240 g of flour. Mark both on the number lines.



2 Divide by 4 to find 1 flat bread: 240 ÷ 4 = 60 g.



3 Multiply by 10 to find 10 flat breads: 60 × 10 = 600 g.

WORKED EXAMPLE

Four flat breads need 240 g of flour. How much flour do 10 flat breads need?

1. 1 flat bread needs 240 ÷ 4 = 60 g.
2. 10 flat breads need 10 × 60 = 600 g.

Answer: 600 g

flat breads	flour (g)
1	60
4	240
10	600

Every row has 60 g of flour for each flat bread.

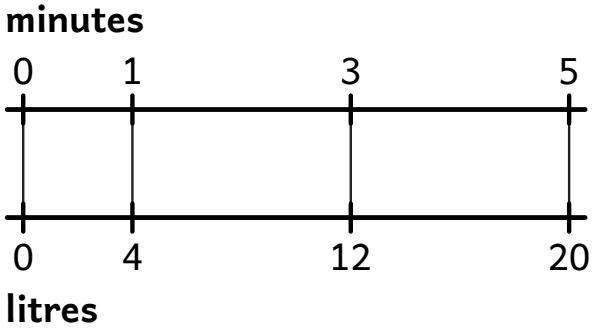
You can also find one first when the question asks for time or for how many. Work out how much one unit gives, then see how many units you need.

WORKED EXAMPLE

A tap fills 12 litres in 3 minutes. How long does it take to fill 20 litres?

1. 1 minute fills 12 ÷ 3 = 4 litres.
2. 20 litres need 20 ÷ 4 = 5 minutes.
3. The tap takes 5 minutes.

Answer: 5 minutes



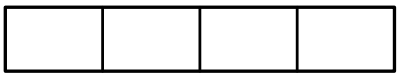
The tap adds 4 litres every minute. 20 litres is 5 lots of 4 litres.

SEE IT THREE WAYS

With things

Put the 240 g of flour into 4 equal bowls. One bowl holds the flour for 1 flat bread. Then count out 10 bowls.

As a picture



240 g in 4 equal parts

In symbols

240 ÷ 4 = 60. Then 60 × 10 = 600.

TRY IT

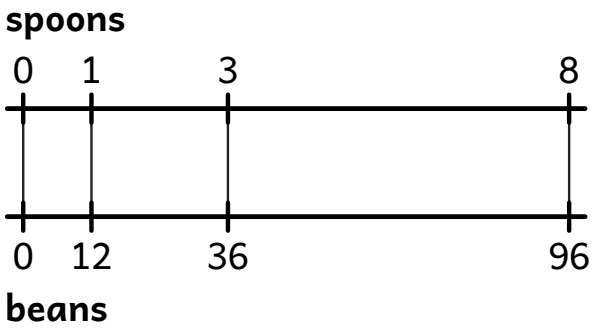
Find the beans in a spoon

You need: a spoon, dry beans or seeds, a bowl.

1. Fill the spoon level with beans. Tip them out and count them. This is the amount for 1 spoon.
2. Now count the beans in 3 level spoons. Check that it is 3 times the first count.
3. Use your count for 1 spoon to guess how many beans are in 8 spoons.
4. Count 8 spoons to test your guess.

What you will see: For example, if 1 spoon holds 12 beans, then 3 spoons hold 36 and 8 spoons hold 96. Your numbers will differ, but the steps are the same.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~If 4 flat breads need 240 g of flour, then 10 flat breads need 240 + 6 = 246 g.~~

Why it feels right: You want 6 more flat breads, so it seems you should add 6 to the flour as well.

The test that breaks it: 1 flat bread needs 60 g. Six more flat breads need 6 × 60 = 360 g more, not 6 g more. And 240 + 360 = 600.

What is true: Do not add the same number to both amounts. Find the amount for one, then multiply or divide.

KEY WORDS

unit amount
the amount for one, such as the flour for 1 flat bread.

scale up
to make more of a recipe, by multiplying.

scale down
to make less of a recipe, by dividing.

BACK TO YOUR GUESS

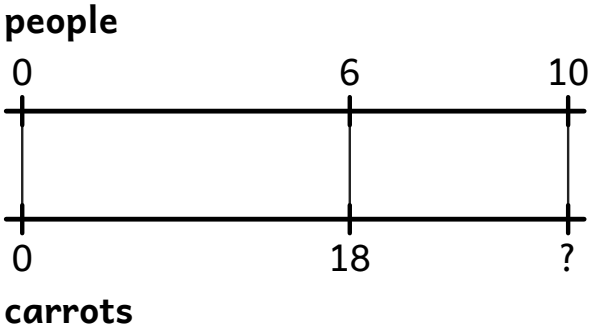
The answer is C: Find the flour for 1 flat bread, then take 10 of those: 600 g.

One flat bread needs 240 ÷ 4 = 60 g of flour. Ten flat breads need 10 × 60 = 600 g. Adding 6 g is far too little. Six more flat breads need much more flour than that.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A soup for 6 people uses 18 carrots. How many carrots does the same soup need for 10 people?



   A 22 carrots

   B 3 carrots

   C 30 carrots

   D 180 carrots

   Answer at the back of the book.

2. 5 shelves hold 120 books. Every shelf holds the same number. How many books fit on 8 shelves?

   A 123 books

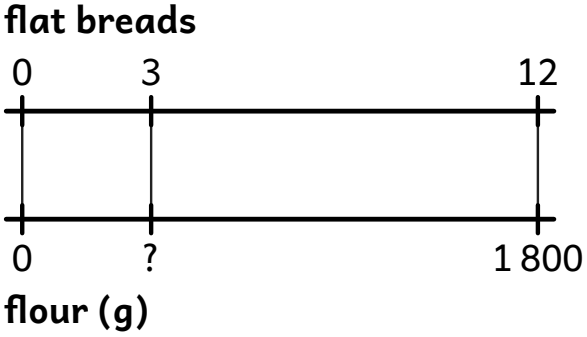
   B 24 books

   C 960 books

   D 192 books

   Answer at the back of the book.

3. A recipe for 12 flat breads uses 1 800 g of flour. How much flour is needed for 3 flat breads?



A 1 791 g

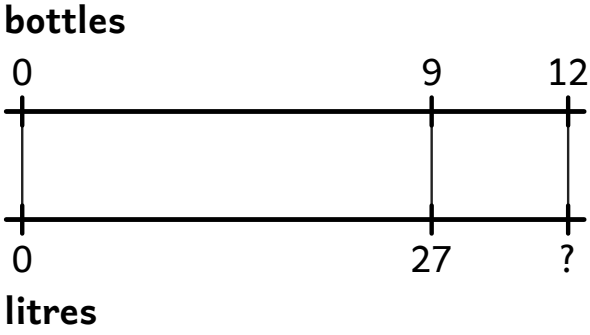
B 150 g

C 600 g

D 450 g

Answer at the back of the book.

4. 9 bottles hold 27 litres in all. All the bottles are the same. How many litres do 12 bottles hold?



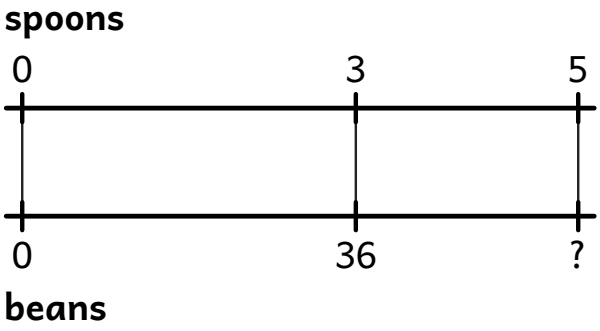
A 36 litres

B 30 litres

C 324 litres

Answer at the back of the book.

5. 3 spoons hold 36 beans. How many beans do 5 spoons hold?



A 41 beans

B 12 beans

C 60 beans

D 180 beans

Answer at the back of the book.

6. A soup for 4 people uses 12 carrots. How many carrots does the same soup need for 9 people?

   Answer at the back of the book.

7. 4 flat breads need 240 g of flour. How much flour do 7 flat breads need?

   Answer at the back of the book.

8. **From an earlier chapter**

   Share 45 seeds between Mei and Yusuf in the ratio 2 : 3. How many seeds does each get?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Find the amount for one by dividing. Then multiply to get the amount you want. Do not add the same number to both amounts. Check with one more example.

Draw: Draw two number lines. Mark 4 and 240 g. Draw an arrow to 1 and 60 g, then an arrow to 10 and 600 g.

M3.4.5 CHAPTER 5

Direct proportion

When one amount doubles, does the other amount always double too?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. If one amount gets bigger, the other gets bigger by the same factor.
- ☐ **B** No. It only happens when the two amounts are in direct proportion.
- ☐ **C** No. Two amounts can never double together.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.8. The try it uses the distance round a square and the area of a square.

BY THE END YOU CAN

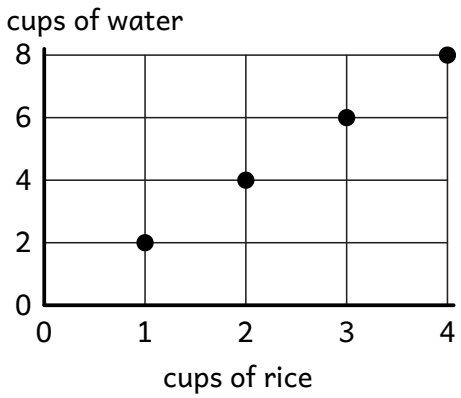
- Spot when two amounts grow at the same rate
- Use a table of values to solve a proportion problem

Two amounts are in **direct proportion** when they grow in step. If one doubles, so does the other. If one is 3 times as big, so is the other.

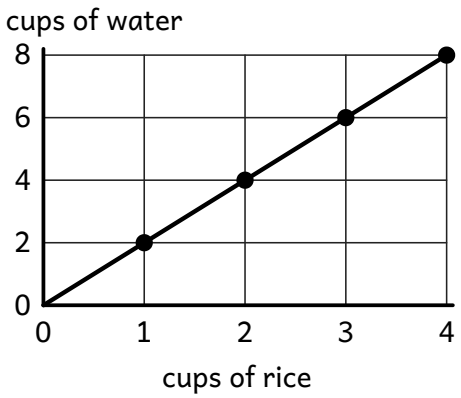
A **table of values** lists pairs of amounts. Divide the second amount by the first in every row. If you always get the same number, the amounts are in direct proportion.

DRAW IT FIRST

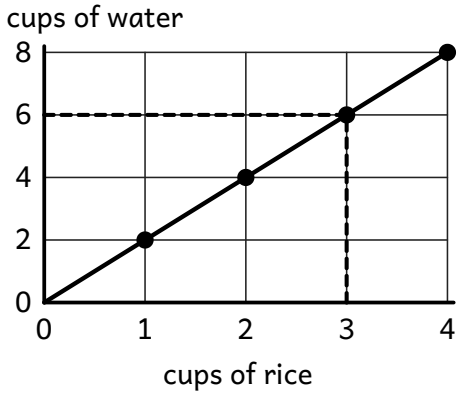
Draw rice and water as a graph



1 Rice needs twice as much water. Mark the pairs 1 and 2, 2 and 4, 3 and 6, 4 and 8.



2 Join the dots. They lie on a straight line. It starts at the origin, the point 0 and 0.



3 Read across from 3 cups of rice. The line gives 6 cups of water.

On a graph, direct proportion gives a straight line through the **origin**. The origin is the point where both amounts are 0. No rice needs no water, so the line must start there.

WORKED EXAMPLE

Beans are weighed by the cup. 2 cups weigh 300 g, 3 cups weigh 450 g and 5 cups weigh 750 g. How much do 8 cups weigh?

1. Divide each weight by its cups: 300 ÷ 2 = 150, 450 ÷ 3 = 150 and 750 ÷ 5 = 150.
2. The answers are all equal, so the weight is in direct proportion to the cups. 1 cup weighs 150 g.
3. 8 cups weigh 8 × 150 = 1 200 g.

Answer: 1 200 g

cups	weight (g)
2	300
3	450
5	750

Weight ÷ cups is 150 g in every row.

SEE IT THREE WAYS

With things

Ravi cooks rice with twice as much water. 1 cup of rice goes with 2 cups of water. 2 cups of rice go with 4 cups of water.

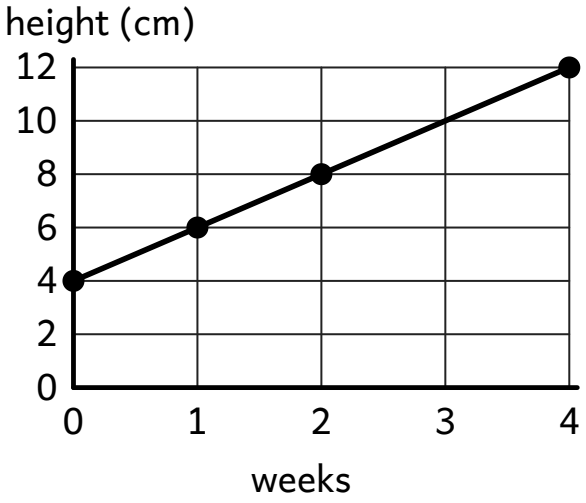
As a picture

rice (cups)	water (cups)
1	2
2	4
3	6
5	10

In symbols

Water ÷ rice is 2 each time: 2 ÷ 1 = 2, 4 ÷ 2 = 2 and 6 ÷ 3 = 2. The ratio of rice to water stays 1 : 2.

Now look at a case that is not direct proportion. Ana plants a seedling that is 4 cm tall. It grows a steady 2 cm each week, but it started at 4 cm, not 0. So when the weeks double, the height does not.



The line starts at 4, not at 0. It does not go through the origin.

WORKED EXAMPLE

Ana's seedling is 4 cm tall. It is 6 cm after 1 week. It is 8 cm after 2 weeks. It is 12 cm after 4 weeks. Is the height in direct proportion to the weeks?

1. Week 1: 6 ÷ 1 = 6.
2. Week 2: 8 ÷ 2 = 4.
3. Week 4: 12 ÷ 4 = 3.
4. The answers 6, 4 and 3 are not equal. So the height is not in direct proportion to the weeks.

Answer: No. The answers 6, 4 and 3 are not equal.

TRY IT

Test a square for direct proportion

You need: squared paper, a pencil.

1. Draw 3 squares on squared paper. Their sides are 1 cm, 2 cm and 3 cm.
2. For each square, count the squares along its edge all the way round. This is the distance round.
3. For each square, count all the small squares inside it. This is the area.
4. Divide the distance round by the side, then the area by the side.

What you will see: Distance round ÷ side is 4 every time, so the distance round is in direct proportion to the side. Area ÷ side is 1, then 2, then 3, so the area is not.

If you cannot try it, this is what happens.

side (cm)	1	2	3
round (cm)	4	8	12
area (cm^2)	1	4	9

A WRONG IDEA MANY PEOPLE HAVE

~~If one amount grows when the other grows, they are in direct proportion.~~

Why it feels right: Often more of one thing goes with more of another. So the idea seems to hold.

The test that breaks it: Look at the seedling. It grew from 8 cm to 12 cm while the weeks went from 2 to 4. The weeks doubled, but the height did not.

What is true: Direct proportion needs the second amount divided by the first to stay the same. The graph must pass through the origin.

KEY WORDS

direct proportion
two amounts that grow in step, so one divided by the other never changes.

table of values
a table that lists pairs of amounts, one pair in each row.

origin
the point on a graph where both amounts are 0.

BACK TO YOUR GUESS

The answer is B: No. It only happens when the two amounts are in direct proportion.

Rice and water double together when the water is always twice the rice. That is direct proportion. A plant that starts 4 cm tall does not double its height when its weeks double. Always test with a table.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A builder lays bricks at an even speed. Are the bricks laid in direct proportion to the hours?

hours	bricks
1	30
2	60
3	90
4	120

A Yes. Bricks ÷ hours is 30 every time.

B Yes. The bricks grow when the hours grow.

C No. The numbers are different in every row.

Answer at the back of the book.

2. A tank has 4 litres in it at the start. A tap adds 1 litre each minute. The litres after 1, 2 and 4 minutes are 5, 6 and 8. Are the litres in direct proportion to the minutes?

minutes	litres
1	5
2	6
4	8

A Yes. Litres ÷ minutes gives the same number.

B No. Litres ÷ minutes gives 5, 3 and 2.

C Yes. The litres go up by 1 each minute.

Answer at the back of the book.

3. Paint is poured at a steady rate. The table shows the litres poured out. How many litres pour out in 9 minutes?

minutes	litres
2	14
3	21
9	?

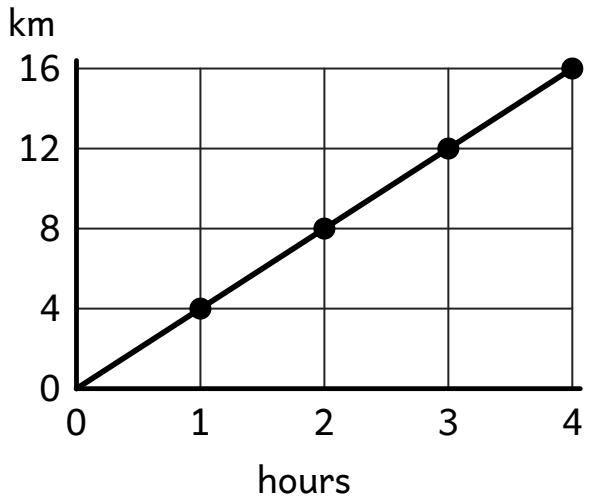
A 30 litres

B 7 litres

C 63 litres

Answer at the back of the book.

4. Leila walks at an even pace. The graph shows how far she walks. How far has she walked after 2.5 hours?



A 8 km

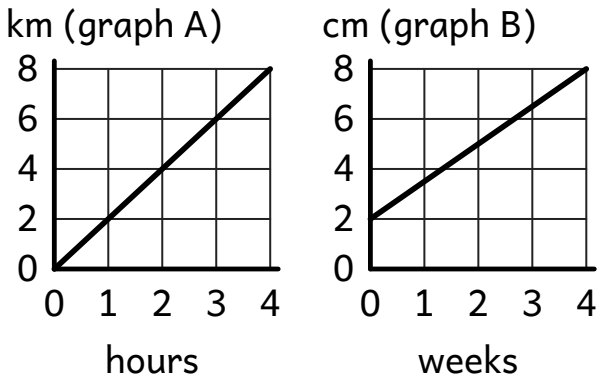
B 9 km

C 10 km

D 12 km

Answer at the back of the book.

5. Which graph shows two amounts in direct proportion?



A Graph A

B Graph B

C Both graphs

Answer at the back of the book.

6. Water pours at a steady rate. In 4 minutes 20 litres pour out, and in 6 minutes 30 litres. How many litres pour out in 9 minutes?

   Answer at the back of the book.

7. Two amounts are in direct proportion. When the first is 4, the second is 12. What is the second when the first is 7?

   Answer at the back of the book.

8. **From an earlier chapter**

   A soup for 8 people uses 24 carrots. How many carrots does the same soup need for 5 people?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

In direct proportion, the second amount divided by the first is the same in every row. The graph is a straight line through the origin. Test every pair, not just one.

Draw: Draw a graph with axes from 0. Plot 1 and 2, 2 and 4, 3 and 6. Join them with a line through the origin.

M3.4.6 CHAPTER 6

Rates and speed

> **Priya walks 6 km in 2 hours. Tariq walks 10 km in 5 hours. Who walks faster?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Tariq, because he walks further.
- ☐ **B** Priya, because she walks 3 km in each hour and Tariq walks 2 km.
- ☐ **C** Tariq, because he walks for longer.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.7. Speed uses hours, and hours and minutes can be changed into each other.

BY THE END YOU CAN

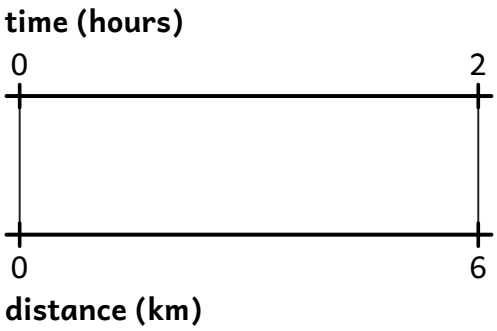
- Work out a speed in km per hour from distance and time
- Find which of two bags gives more rice for each coin

A **rate** compares two different kinds of amount, such as kilometres and hours. Speed is a rate.

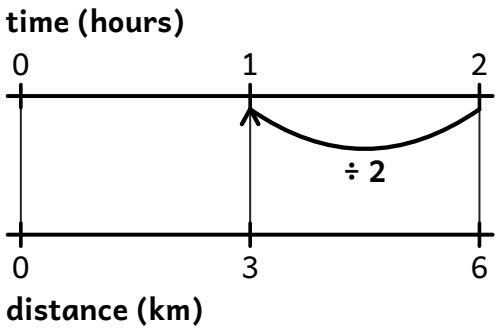
Speed tells you how far you go in each unit of time. A speed in km per hour is the distance in one hour. A speed of 3 km per hour means 3 km in each hour. The word **per** means for each.

DRAW IT FIRST

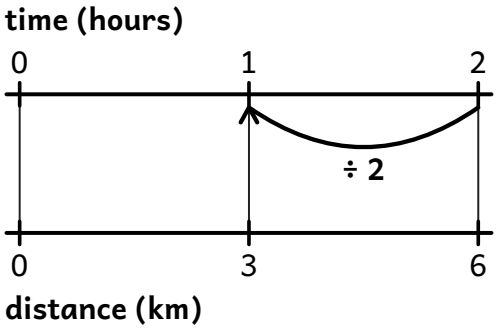
Find the distance for one hour



1 Priya walks 6 km in 2 hours. Mark both on the number lines.



2 Divide by 2 to find 1 hour: 6 ÷ 2 = 3 km.



speed: 3 km per hour

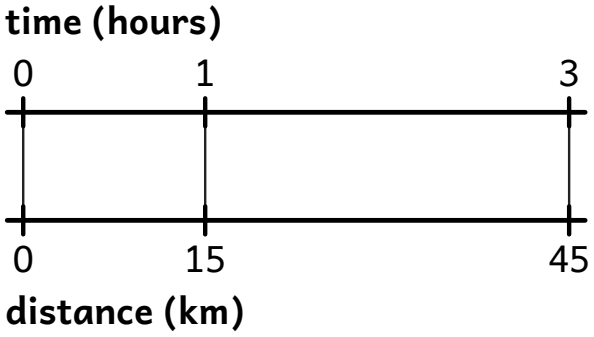
3 Priya walks 3 km in each hour. Her speed is 3 km per hour.

WORKED EXAMPLE

Kofi cycles 45 km in 3 hours. What is his speed?

1. Speed is distance ÷ time.
2. 45 ÷ 3 = 15.
3. His speed is 15 km per hour.

Answer: 15 km per hour



Kofi goes 15 km in each hour, so 3 hours is 45 km.

If you know the speed, you can find the distance. Distance = speed × time. You can also find the time. Time = distance ÷ speed.

Rates also help you choose the better buy. Find how much you get for 1 coin. The bag with more rice for 1 coin is the better buy.

WORKED EXAMPLE

Bag A holds 5 kg of rice for 2 coins. Bag B holds 9 kg of rice for 4 coins. Which bag gives more rice for each coin?

1. Bag A, rice for 1 coin: 5 ÷ 2 = 2.5 kg.
2. Bag B, rice for 1 coin: 9 ÷ 4 = 2.25 kg.
3. Compare: 2.5 is more than 2.25, so bag A gives more rice for each coin.

Answer: Bag A is the better buy. It gives 2.5 kg for each coin. Bag B gives 2.25 kg.

bag	rice (kg)	coins
A	5	2
B	9	4

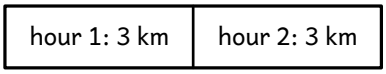
Bag B is bigger. But look at the rice for each coin.

SEE IT THREE WAYS

With things

Priya walks 6 km in 2 hours. Put 6 stones in 2 equal piles, one pile for each hour.

As a picture



6 km in 2 hours

In symbols

6 km ÷ 2 hours = 3 km per hour.

TRY IT

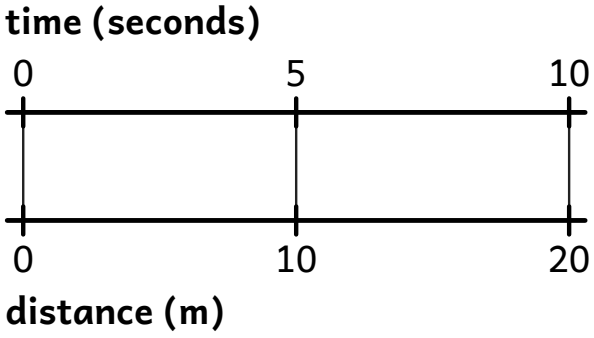
Find your own walking speed

You need: a piece of string 10 m long, or a tape measure, a friend who can count seconds.

1. Lay the string on the ground to mark 10 m.
2. Walk along it at a steady pace. Your friend counts the seconds out loud.
3. Divide 10 by the number of seconds. This is your speed in metres per second.
4. Walk it again, faster. Compare the two speeds.

What you will see: For example, 10 m in 5 seconds is 10 ÷ 5 = 2 metres each second. A faster walk takes fewer seconds, so its speed number is bigger.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The bigger bag always gives more rice for each coin.~~

Why it feels right: A bigger bag holds more rice, so it looks like the better buy.

The test that breaks it: Bag C holds 12 kg for 6 coins. That is 12 ÷ 6 = 2 kg for each coin. Bag D holds 15 kg for 10 coins. That is 15 ÷ 10 = 1.5 kg for each coin. Bag D is bigger, but bag C gives more for each coin.

What is true: Divide the amount by the number of coins for each bag. The bag with the bigger answer is the better buy.

KEY WORDS

rate
a comparison of two different kinds of amount, such as km and hours.

speed
the distance you go in each hour, or each second, such as 15 km per hour.

per
for each, as in 3 km per hour.

BACK TO YOUR GUESS

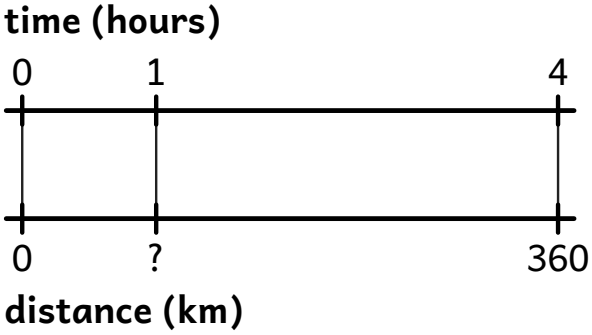
The answer is B: Priya, because she walks 3 km in each hour and Tariq walks 2 km.

To compare speeds, find the distance for 1 hour. Priya walks 6 ÷ 2 = 3 km each hour. Tariq walks 10 ÷ 5 = 2 km each hour. Priya is faster, even though Tariq goes further.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A train travels 360 km in 4 hours. What is its speed in km per hour?



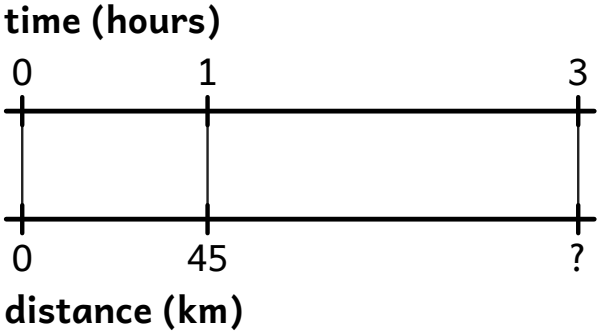
A 1 440 km per hour

B 90 km per hour

C 356 km per hour

Answer at the back of the book.

2. A bus drives at 45 km per hour for 3 hours. How far does it go?



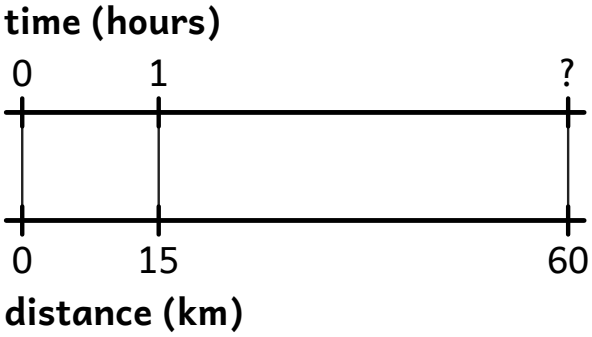
A 48 km

B 15 km

C 135 km

Answer at the back of the book.

3. A cyclist rides at 15 km per hour. How many hours does it take to ride 60 km?



A 900 hours

B 45 hours

C 4 hours

Answer at the back of the book.

4. Bag P holds 9 kg of rice for 3 coins. Bag Q holds 14 kg of rice for 7 coins. Which bag gives more rice for each coin?

bag	rice (kg)	coins
P	9	3
Q	14	7

A Bag Q

B Bag P

C They give the same

Answer at the back of the book.

5. Priya walks 12 km in 3 hours. Tariq walks 10 km in 2 hours. Who walks faster?

name	km	hours
Priya	12	3
Tariq	10	2

A Priya, because 12 km is further than 10 km.

B Tariq, at 5 km per hour. Priya goes 4 km per hour.

C They walk at the same speed.

Answer at the back of the book.

6. A cyclist rides 60 km in 4 hours. What is the speed in km per hour?

Answer at the back of the book.

7. A walker goes at 4 km per hour. How many hours does it take to walk 20 km?

Answer at the back of the book.

8. **From an earlier chapter**

Two amounts are in direct proportion. When the first is 5, the second is 15. What is the second when the first is 8?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Speed is distance ÷ time, in km per hour. Distance = speed × time. For the better buy, find the amount for 1 coin. The bigger bag is not always better.

Draw: Draw two number lines. Mark 2 hours and 6 km. Draw an arrow to 1 hour and 3 km. Write 3 km per hour.

M3.4.7 CHAPTER 7

Maps and scale drawings

> **On a map a road is 4 cm long. The scale is 1 : 50 000. How long is the real road?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The real road is 4 cm long, the same as on the map.
- ☐ **B** The real road is 2 km long.
- ☐ **C** The real road is 200 km long.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.2. Change centimetres to metres and kilometres: 100 cm make 1 m, and 1 000 m make 1 km.

BY THE END YOU CAN

- Use a scale of 1 : 50 000 to find a real distance
- Draw a plan of a room with 1 cm for each 1 m

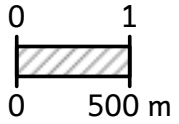
A map shows a big place on a small page. The **scale** tells you how much smaller the map is. A scale of 1 : 50 000 means this. 1 cm on the map is 50 000 cm on the ground.

50 000 cm is hard to picture. Change it to metres. 100 cm make 1 m, so 50 000 ÷ 100 = 500 m. So on this map 1 cm stands for 500 m. A **scale bar** shows this on the map itself.

DRAW IT FIRST

Draw a scale bar for 1 : 50 000

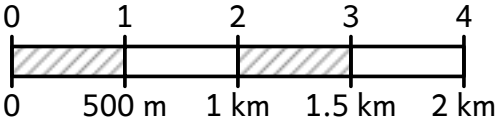
on the map (cm)



on the ground

1 Draw 1 cm on the map. On the ground it is 50 000 cm, which is 500 m.

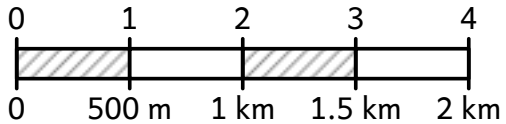
on the map (cm)



on the ground

2 Add more centimetres. Each one stands for another 500 m, so 2 cm stand for 1 km.

on the map (cm)



on the ground

1 cm stands for 500 m

3 Read the bar: 1 cm on the map stands for 500 m on the ground.

WORKED EXAMPLE

A map has a scale of 1 : 50 000. A path is 6 cm long on the map. How far is it on the ground, in km?

1. 1 cm is 50 000 cm, so 6 cm is 6 × 50 000 = 300 000 cm.
2. Change to metres: 300 000 ÷ 100 = 3 000 m.
3. Change to kilometres: 3 000 ÷ 1 000 = 3 km.

Answer: 3 km

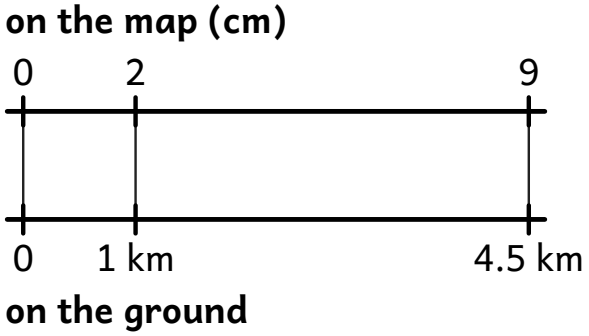
You can also go the other way. Change the real distance to centimetres. Then divide by the scale to find the length on the map.

WORKED EXAMPLE

Two villages are 4.5 km apart. The map has a scale of 1 : 50 000. How far apart are they on the map?

1. Change to centimetres: 4.5 km = 4 500 m = 450 000 cm.
2. Divide by the scale: 450 000 ÷ 50 000 = 9.
3. On the map they are 9 cm apart.

Answer: 9 cm



Every 2 cm on the map is 1 km on the ground. The villages are 4.5 km apart, so they are 9 cm apart.

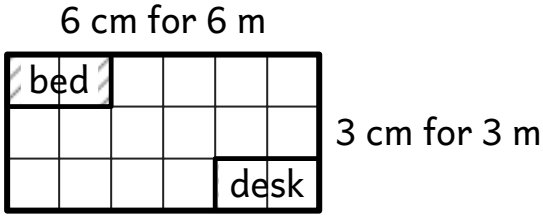
A plan is a scale drawing of a room. Choose a scale that fits your page. With 1 cm for each 1 m, a wall 6 m long is drawn 6 cm long. This scale is 1 : 100, because 1 m is 100 cm.

WORKED EXAMPLE

A room is 6 m long and 3 m wide. A bed is 2 m long. Draw a plan with 1 cm for each 1 m.

1. The scale is 1 cm for each 1 m.
2. The room is 6 m long. 6 ÷ 1 = 6 cm.
3. The room is 3 m wide. 3 ÷ 1 = 3 cm.
4. The bed is 2 m long. 2 ÷ 1 = 2 cm.
5. Draw a rectangle 6 cm by 3 cm. Draw the bed 2 cm long inside it.

Answer: 6 cm by 3 cm



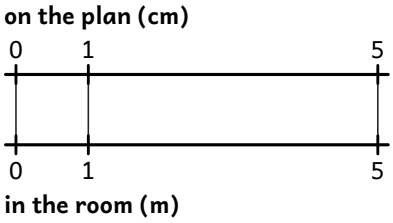
Each square is 1 cm on the plan. It stands for 1 m in the room.

SEE IT THREE WAYS

With things

Draw a line 1 cm long and say it is 1 m. A line 5 cm long is then 5 m.

As a picture



In symbols

1 cm on the plan is 1 m in the room, so 5 cm is 5 m. The scale is 1 : 100.

TRY IT

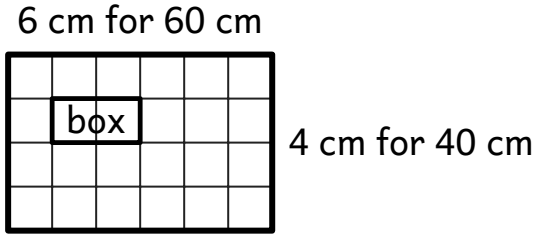
Draw a plan of a table

You need: squared paper, a ruler, a table or a desk, and a tape measure or a piece of string.

1. Measure the length and the width of the table top in centimetres.
2. Choose a scale. Use 1 cm on the paper for each 10 cm on the table.
3. Divide each measurement by 10 to get the lengths to draw.
4. Draw the rectangle on squared paper. Draw a box on it, to the same scale.

What you will see: For example, a table 60 cm by 40 cm is drawn 6 cm by 4 cm. A box 20 cm by 10 cm is drawn 2 cm by 1 cm.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~On a map with the scale 1 : 50 000, 1 cm stands for 50 000 m.~~

Why it feels right: The number 50 000 is big, and it is quick to forget which unit it is in.

The test that breaks it: Both numbers in 1 : 50 000 must be in the same unit. If 1 cm stood for 50 000 m, that is 5 000 000 cm. The scale would then be 1 : 5 000 000, which is not the scale we have.

What is true: In the scale 1 : 50 000 both numbers use the same unit. So 1 cm on the map is 50 000 cm on the ground. That is 500 m.

KEY WORDS

scale

how much smaller a map or plan is than the real place, such as 1 : 50 000.

scale bar
a line on a map that shows what a length on the map stands for.

plan
a scale drawing of a room or a building, seen from above.

BACK TO YOUR GUESS

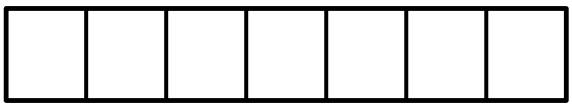
The answer is B: The real road is 2 km long.

1 cm on the map is 50 000 cm on the ground. 4 cm is 4 × 50 000 = 200 000 cm. That is 2 000 m, which is 2 km. If you read 200 000 as metres you get 200 km, but the scale is in centimetres.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A map has the scale 1 : 50 000. A road is 7 cm long on the map. How long is the real road?



road: 7 cm on the map

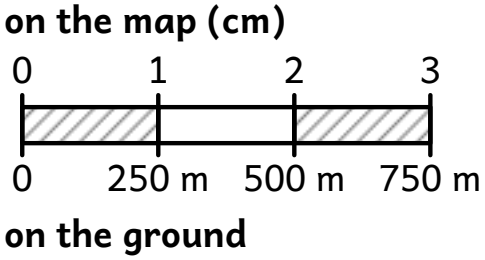
A 350 km

B 3.5 km

C 350 m

Answer at the back of the book.

2. A map has the scale 1 : 25 000. A path is 6 cm long on the map. How long is the real path?



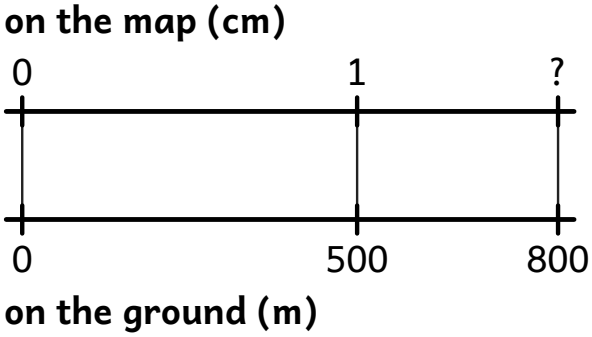
A 150 km

B 150 m

C 1.5 km

Answer at the back of the book.

3. A field is 800 m long. The scale of the map is 1 : 50 000. How long is the field on the map?



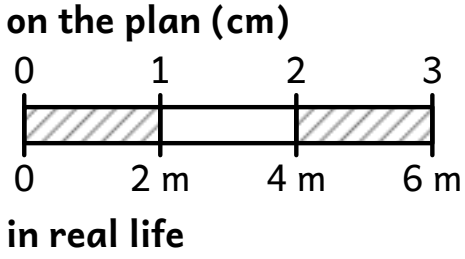
A 1.6 cm

B 16 cm

C 40 000 000 cm

Answer at the back of the book.

4. Draw a plan of a hall 12 m long and 8 m wide. Use 1 cm for each 2 m. What size is the plan?



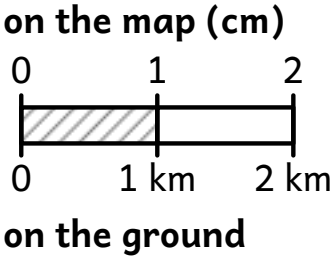
A 12 cm by 8 cm

B 6 cm by 4 cm

C 24 cm by 16 cm

Answer at the back of the book.

5. Which scale means that 1 cm on the map stands for 1 km on the ground?



A 1 : 1 000

B 1 : 100 000

C 1 : 1 000 000

Answer at the back of the book.

6. A map has the scale 1 : 50 000. Two towns are 9 cm apart on the map. How many km apart are they?

Answer at the back of the book.

7. A room is 7 m long and 5 m wide. How long and wide is its plan, if 1 cm stands for 1 m?

Answer at the back of the book.

8. **From an earlier chapter**

A bike goes 36 km in 3 hours. What is its speed in km per hour?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

On a 1 : 50 000 map, 1 cm is 50 000 cm on the ground. That is 500 m. Multiply to go from map to ground. Divide to go back.

Draw: Draw a scale bar of 4 cm. Mark 0, 500 m, 1 km, 1.5 km and 2 km under the ticks.

M3.4.8 CHAPTER 8

Inverse proportion

> **3 people dig a ditch in 8 days. How long would 6 people take?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 16 days, because there are twice as many people.
- ☐ **B** 4 days, because twice as many people finish in half the time.
- ☐ **C** 5 days, because 6 people is 3 more than 3 people.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.6. Inverse proportion uses times and divide facts, such as $3 \times 8 = 24$ and $24 \div 6 = 4$.

BY THE END YOU CAN

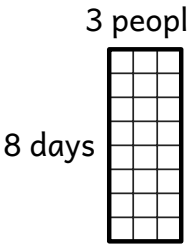
- Spot when one amount goes down as the other goes up
- Find how long 6 people take if 3 people take 8 days

Sometimes more of one amount means less of another. When more people share a job, it takes fewer days. If the two amounts always multiply to the same number, this is **inverse proportion**. Then when one amount doubles, the other halves.

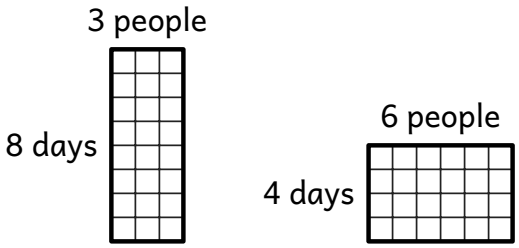
Measure the work in **person-days**. One person working for one day does 1 person-day. 3 people working for 8 days do $3 \times 8 = 24$ person-days.

DRAW IT FIRST

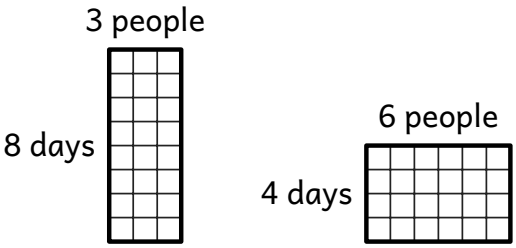
Draw the same job two ways



1 Each square is 1 person-day. 3 people for 8 days make a rectangle of 3 by 8 squares.



2 6 people for 4 days make a rectangle of 6 by 4 squares. It is a different shape.



3 × 8 = 24 and 6 × 4 = 24

3 Count the squares in each. Both rectangles have 24 squares, so both are the same job.

WORKED EXAMPLE

3 people dig a ditch in 8 days. How long would 6 people take?

1. 3 people for 8 days is 3 × 8 = 24 person-days.
2. Everyone works at the same speed, so the job is always 24 person-days.
3. With 6 people, each person works 24 ÷ 6 = 4 days.

Answer: 4 days

The job stays the same size, so the person-days stay the same. In inverse proportion the two amounts multiply to the same number every time.

This works only when every person works at the same speed, and the job can be shared out.

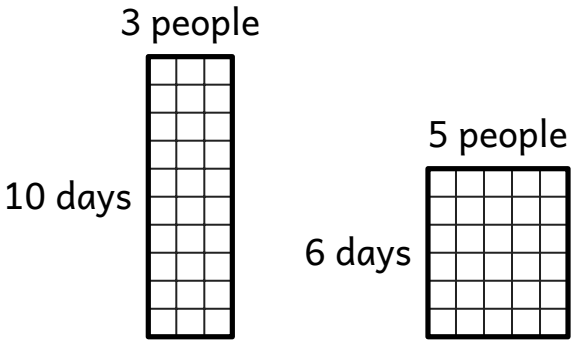
Going down is not enough. A candle burns for more hours and gets shorter, but hours × height does not stay the same.

WORKED EXAMPLE

3 painters paint a long wall in 10 days. How long would 5 painters take?

1. 3 painters for 10 days is 3 × 10 = 30 person-days.
2. Everyone works at the same speed, so the wall is always 30 person-days of work.
3. With 5 painters, each works 30 ÷ 5 = 6 days.

Answer: 6 days



Each square is 1 person-day. Both rectangles have 30 squares. Here the people are painters.

SEE IT THREE WAYS

With things

Take 24 stones. Each stone is 1 person-day. Make equal rows. Try 2 rows of 12, then 3 of 8, 4 of 6 and 6 of 4.

As a picture

people	days
1	24
2	12
3	8
4	6
6	4
8	3

In symbols

people × days = 24 every time. 1 × 24 = 2 × 12 = 3 × 8 = 4 × 6 = 6 × 4 = 8 × 3 = 24.

TRY IT

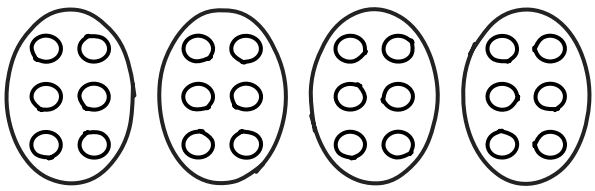
Share 24 stones into equal groups

You need: 24 small stones, or beans.

1. Make 2 equal groups. Count the stones in one group.
2. Mix them up. Make 3 equal groups, then 4, then 6, and count each time.
3. Write each pair in a table: the number of groups and the stones in each group.
4. Multiply the two numbers in each row.

What you will see: More groups means fewer stones in each group. The two numbers multiply to 24 every time, so they are in inverse proportion.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~If 3 people take 8 days, then 6 people take 16 days, because 6 is double 3.~~

Why it feels right: When one amount doubles, the other doubles too in many problems, such as rice and water.

The test that breaks it: 6 people working for 16 days do 6 × 16 = 96 person-days. The job is only 24 person-days. That is four times too much work.

What is true: More people do the job faster. When the people double, the days halve.

KEY WORDS

inverse proportion
two amounts that always multiply to the same number, so one goes down as the other goes up.

person-days
the size of a job: the number of people times the number of days.

BACK TO YOUR GUESS

The answer is B: 4 days, because twice as many people finish in half the time.

More people share the digging, so each one works fewer days. 3 people for 8 days is 24 person-days of work. 6 people share 24 person-days, so each works 24 ÷ 6 = 4 days.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A job takes 6 people 10 days. How many days would 4 people take?

people	days
6	10
4	?

A 8 days

B 40 days

C 15 days

Answer at the back of the book.

2. A trip takes 3 hours at 60 km per hour. How long does the same trip take at 90 km per hour?

km per hour	hours
60	3
90	?

A 4.5 hours

B 1.5 hours

C 2 hours

Answer at the back of the book.

3. Which pair of amounts is in inverse proportion?

A The number of notebooks and their total weight

B The number of people sharing a cake and the size of each slice

C The hours a candle burns and its height

Answer at the back of the book.

4. The same job is done by different numbers of people. Find the missing number of people.

people	days
1	24
2	12
4	6
?	3

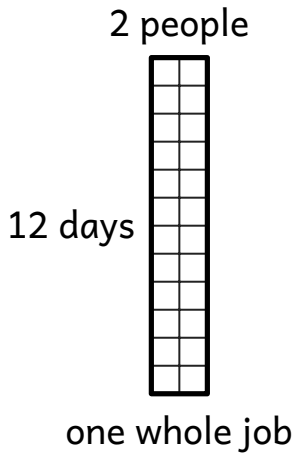
A 6 people

B 8 people

C 12 people

Answer at the back of the book.

5. A job takes 2 people 12 days. How many days would 8 people take?



A 48 days

B 6 days

C 96 days

D 3 days

Answer at the back of the book.

6. 4 people build a wall in 9 days. How many days would 6 people take?

Answer at the back of the book.

7. A tank is emptied by 5 pumps in 6 hours. How many hours would 10 pumps take?

   Answer at the back of the book.

8. **From an earlier chapter**

   A map has the scale 1 : 50 000. Two towns are 8 cm apart on the map. How many km apart are they?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

In inverse proportion one amount goes down as the other goes up. Their product stays the same. Find the job in person-days, then divide by the new number of people. Everyone must work at the same speed.

Draw: Draw a rectangle 3 squares across and 8 down. Draw another 6 across and 4 down. Write 24 in each.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

direct proportion
two amounts that grow in step, so one divided by the other never changes. M3.4.5

equal ratios
ratios that describe the same mix, such as 6 : 4 and 3 : 2. M3.4.1

inverse proportion
two amounts that always multiply to the same number, so one goes down as the other goes up. M3.4.8

one part
the amount that each equal part of the ratio holds. M3.4.3

origin
the point on a graph where both amounts are 0. M3.4.5

part to part
a ratio, which compares one part with another part. M3.4.2

part to whole
a fraction, which compares one part with all the parts. M3.4.2

parts
the equal pieces that the numbers of a ratio count. M3.4.2

per
for each, as in 3 km per hour. M3.4.6

person-days
the size of a job: the number of people times the number of days. M3.4.8

plan
a scale drawing of a room or a building, seen from above. M3.4.7

rate
a comparison of two different kinds of amount, such as km and hours. M3.4.6

ratio
a way to compare two amounts of the same kind, written like 6 : 4. M3.4.1

scale
how much smaller a map or plan is than the real place, such as 1 : 50 000. M3.4.7

scale bar

a line on a map that shows what a length on the map stands for. M3.4.7

scale down
to make less of a recipe, by dividing. M3.4.4

scale up
to make more of a recipe, by multiplying. M3.4.4

simplest form
a ratio where only 1 goes into both numbers. M3.4.1

speed
the distance you go in each hour, or each second, such as 15 km per hour. M3.4.6

table of values
a table that lists pairs of amounts, one pair in each row. M3.4.5

total
the whole amount that is shared. M3.4.3

unit amount
the amount for one, such as the flour for 1 flat bread. M3.4.4

Answers

M3.4.1. Ratio: comparing parts

1. **5 : 3**

   Count the filled dots: 5. Count the empty dots: 3. Mangoes come first, so the ratio is 5 : 3.

   - **If you chose A:** The order is flipped. The question asks for mangoes to limes, so the mangoes come first. Go to M3.4.1.
   - **If you chose C:** You compared the mangoes with all the fruit. A ratio compares one part with another part, so count the limes alone. Go to M3.4.1.

2. **3 : 5**

   The highest common factor of 15 and 25 is 5. 15 ÷ 5 = 3 and 25 ÷ 5 = 5.

   - **If you chose B:** The numbers are right but the order is flipped. A ratio keeps its order when you simplify. Go to M3.4.1.
   - **If you chose C:** You divided only the first number. Divide both numbers by the same number. Go to M3.4.1.

3. **2 : 3**

   Divide both numbers by 2: 4 ÷ 2 = 2 and 6 ÷ 2 = 3. So 4 : 6 = 2 : 3.

   - **If you chose A:** You added 2 to each number. Adding does not keep the mix the same. Divide or multiply both numbers instead. Go to M3.4.1.
   - **If you chose C:** Only the second number changed, so the mix changed. Both numbers must change by the same factor. Go to M3.4.1.

4. **5 : 1**

   Make the units match: 3 m = 300 cm. The ratio is 300 : 60. The highest common factor is 60. 300 ÷ 60 = 5 and 60 ÷ 60 = 1.

   - **If you chose A:** The units do not match. 3 is in metres and 60 is in centimetres. Change 3 m to 300 cm first. Go to M3.4.1.
   - **If you chose B:** That is 3 : 60 simplified, but 3 m and 60 cm are not the same unit. Make the units match before you simplify. Go to M3.4.1.

5. **3 : 2**

   The highest common factor of 12 and 8 is 4. 12 ÷ 4 = 3 and 8 ÷ 4 = 2. Only 1 goes into both 3 and 2.

   - **If you chose A:** This is a correct ratio, but it is not in simplest form. Both numbers can still be divided by 4. Go to M3.4.1.
   - **If you chose B:** This is equal to 12 : 8, but both numbers can still be divided by 2. Keep going until only 1 goes into both. Go to M3.4.1.

6. **3 : 4**

   The highest common factor of 18 and 24 is 6. 18 ÷ 6 = 3 and 24 ÷ 6 = 4.

7. **8 : 1**

   Make the units match: 4 m = 400 cm. The ratio is 400 : 50. The highest common factor is 50. 400 ÷ 50 = 8 and 50 ÷ 50 = 1.

M3.4.2. Ratios and fractions

1. **$^1/_5$**

   1 + 4 = 5 parts in all. Goats are 1 of the 5 parts.

   - **If you chose A:** That compares goats with sheep only. The whole flock is goats and sheep together: 1 + 4 = 5 parts. Go to M3.4.2.
   - **If you chose C:** That is the fraction of sheep. The goats are 1 of the 5 parts. Go to M3.4.2.

2. **$^3/_8$**

   5 + 3 = 8 parts in all. Cats are 3 of the 8 parts.

   - **If you chose A:** That compares cats with dogs. The whole is dogs and cats together: 5 + 3 = 8 parts. Go to M3.4.2.
   - **If you chose B:** That is the fraction of dogs. The question asks about cats, which are 3 of the 8 parts. Go to M3.4.2.

3. **2 : 5**

   Out of 7 parts, 2 are walkers. The other 7 − 2 = 5 parts come by bus. The ratio is 2 : 5.

   - **If you chose A:** The 7 is the whole class. Bus riders are the parts that are left: 7 − 2 = 5. Go to M3.4.2.
   - **If you chose B:** The numbers are right but the order is flipped. Walkers come first. Go to M3.4.2.

4. **3 : 5**

   3 parts are shaded and 5 are plain. Shaded comes first, so the ratio is 3 : 5.

   - **If you chose A:** You counted all the parts in the bar. That makes a fraction, 3 out of 8. A ratio compares shaded with plain. Go to M3.4.2.
   - **If you chose C:** The order is flipped. Shaded comes first, so write the shaded count first. Go to M3.4.2.

5. **Juice is $^{2}/_{5}$ of the drink.**

   A fraction compares a part with the whole. The whole drink is 2 + 3 = 5 parts, and juice is 2 of them.

   - **If you chose A:** That uses the 3 from the ratio, but 3 is the water only. The whole drink is 2 + 3 = 5 parts. Go to M3.4.2.
   - **If you chose B:** This is true, but it compares part with part, juice with water. The question asks for part to whole. Go to M3.4.2.

6. $^{3}/_{7}$

   3 + 4 = 7 parts in all. Red is 3 of the 7 parts.

7. **3 : 5**

   Out of 8 parts, 3 are walkers. The other 8 − 3 = 5 parts come by bus. The ratio is 3 : 5.

8. **3 : 2**

   The highest common factor of 18 and 12 is 6. 18 ÷ 6 = 3 and 12 ÷ 6 = 2.

M3.4.3. Sharing in a ratio

1. **15 and 25**

   3 + 5 = 8 parts. 40 ÷ 8 = 5 beans a part. Sipho gets 3 × 5 = 15 and Lucía gets 5 × 5 = 25.

   - **If you chose A:** The numbers in a ratio are parts, not beans. There are 3 + 5 = 8 parts, and 40 beans to share out. Go to M3.4.3.
   - **If you chose B:** That is an equal share. The ratio 3 : 5 gives Lucía more than Sipho. Go to M3.4.3.
   - **If you chose C:** You treated 3 : 5 like the fractions $^{3}/_{5}$ and $^{2}/_{5}$. But the whole is 3 + 5 = 8 parts, not 5. Go to M3.4.3.

2. **24 cm and 30 cm**

   4 + 5 = 9 parts. 54 ÷ 9 = 6 cm a part. The pieces are 4 × 6 = 24 cm and 5 × 6 = 30 cm.

   - **If you chose A:** That cuts the rope in half. The ratio 4 : 5 makes the pieces different lengths. Go to M3.4.3.
   - **If you chose B:** That is 4 × 9 and 5 × 9. 9 is the number of parts. You must divide 54 by 9 first to find one part. Go to M3.4.3.

3. **35 stones**

   Hana has 2 parts, which is 14 stones. One part is 14 ÷ 2 = 7. There are 2 + 3 = 5 parts, so 5 × 7 = 35 stones.

   - **If you chose A:** That is Joseph's share, 3 parts of 7. The question asks for all the stones, Hana's and Joseph's together. Go to M3.4.3.
   - **If you chose B:** That doubles Hana's 14, as if they shared equally. Joseph has 3 parts and Hana has only 2. Go to M3.4.3.

4. **12 sweets**

   3 + 4 = 7 parts. 28 ÷ 7 = 4 sweets a part. The share with 3 parts is 3 × 4 = 12 sweets.

   - **If you chose A:** That is the size of one part, 28 ÷ 7. The share has 3 parts, so multiply by 3. Go to M3.4.3.
   - **If you chose B:** That is the share with 4 parts. The question asks about the share with 3 parts. Go to M3.4.3.
   - **If you chose C:** That is 3 × 7. The 7 is the number of parts, not the size of a part. Divide 28 by 7 first. Go to M3.4.3.

5. **8 litres**

   2 + 7 = 9 parts. 36 ÷ 9 = 4 litres a part. Red has 2 parts: 2 × 4 = 8 litres.

   - **If you chose B:** That is the size of one part. Red has 2 parts, so multiply 4 by 2. Go to M3.4.3.
   - **If you chose C:** That is half of 36. Red and white are not shared equally. Red has 2 parts out of 9. Go to M3.4.3.
   - **If you chose D:** That is the white paint: 7 parts of 4 litres each. Red is the smaller share. Go to M3.4.3.

6. **14 cm and 49 cm**

   2 + 7 = 9 parts. 63 ÷ 9 = 7 cm a part. The pieces are 2 × 7 = 14 cm and 7 × 7 = 49 cm.

7. **40 seeds**

   Ana has 3 parts, which is 15 seeds. One part is 15 ÷ 3 = 5. There are 3 + 5 = 8 parts, so 8 × 5 = 40 seeds.

8. **$^{3}/_{10}$**

   3 + 7 = 10 parts in all. Black is 3 of the 10 parts.

M3.4.4. Finding one first

1. **30 carrots**

   1 person needs 18 ÷ 6 = 3 carrots. 10 people need 10 × 3 = 30 carrots.

   - **If you chose A:** You added 4 because 10 is 4 more than 6. But each extra person needs 3 carrots, so 4 more people need 12 more carrots. Go to M3.4.4.
   - **If you chose B:** That is the carrots for 1 person, which is the first step. Now multiply by 10 people. Go to M3.4.4.
   - **If you chose D:** You multiplied 18 by 10. But 18 carrots are for 6 people, not for 1. Find the carrots for 1 person first. Go to M3.4.4.

2. **192 books**

   1 shelf holds 120 ÷ 5 = 24 books. 8 shelves hold 8 × 24 = 192 books.

   - **If you chose A:** You added 3 because 8 is 3 more than 5. But each shelf holds 24 books, so 3 more shelves hold 72 more. Go to M3.4.4.
   - **If you chose B:** That is the books on 1 shelf. You still need to multiply by 8 shelves. Go to M3.4.4.
   - **If you chose C:** You multiplied 120 by 8. But 120 books are on 5 shelves, not 1. Find the books on 1 shelf first. Go to M3.4.4.

3. **450 g**

   1 flat bread needs 1 800 ÷ 12 = 150 g. 3 flat breads need 3 × 150 = 450 g. This time you scale down.

   - **If you chose A:** You took away 9 because 3 is 9 fewer than 12. But 9 fewer flat breads need far more than 9 g less flour. Go to M3.4.4.
   - **If you chose B:** That is the flour for 1 flat bread. You want 3 flat breads, so multiply by 3. Go to M3.4.4.
   - **If you chose C:** You divided by 3, the number of flat breads you want. Divide by 12 first, the number the recipe is for. Go to M3.4.4.

4. **36 litres**

   1 bottle holds 27 ÷ 9 = 3 litres. 12 bottles hold 12 × 3 = 36 litres.

   - **If you chose B:** You added 3 because 12 is 3 more than 9. But 3 more bottles hold 3 × 3 = 9 litres more, not 3. Go to M3.4.4.
   - **If you chose C:** You multiplied 27 by 12. But 27 litres is for 9 bottles. Find the litres for 1 bottle first. Go to M3.4.4.

5. **60 beans**

   1 spoon holds 36 ÷ 3 = 12 beans. 5 spoons hold 5 × 12 = 60 beans.

   - **If you chose A:** You added 5 to 36. But beans are not added to spoons. Each spoon holds 12 beans, so find that first. Go to M3.4.4.
   - **If you chose B:** That is the beans in 1 spoon. The question asks about 5 spoons, so multiply by 5. Go to M3.4.4.
   - **If you chose D:** You multiplied 36 by 5. But 36 beans are for 3 spoons. Divide by 3 first to find the beans in 1 spoon. Go to M3.4.4.

6. **27 carrots**

   1 person needs 12 ÷ 4 = 3 carrots. 9 people need 9 × 3 = 27 carrots.

7. **420 g**

   1 flat bread needs 240 ÷ 4 = 60 g. 7 flat breads need 7 × 60 = 420 g.

8. **18 and 27**

   2 + 3 = 5 parts. 45 ÷ 5 = 9 seeds a part. Mei gets 2 × 9 = 18 and Yusuf gets 3 × 9 = 27.

M3.4.5. Direct proportion

1. **Yes. Bricks ÷ hours is 30 every time.**

   30 ÷ 1 = 30, 60 ÷ 2 = 30, 90 ÷ 3 = 30 and 120 ÷ 4 = 30. The answers are all equal.

   - **If you chose B:** Growing together is not enough. The seedling grew too. Divide the bricks by the hours in each row. Is the answer the same? Go to M3.4.5.
   - **If you chose C:** The numbers do change, but look at the ratio. Divide each row: 30 ÷ 1, 60 ÷ 2, 90 ÷ 3. What do you get? Go to M3.4.5.

2. **No. Litres ÷ minutes gives 5, 3 and 2.**

   5 ÷ 1 = 5, 6 ÷ 2 = 3 and 8 ÷ 4 = 2. The answers are not equal. A graph would start at 4 litres, not at the origin.

   - **If you chose A:** Check by dividing: 5 ÷ 1 = 5, but 6 ÷ 2 = 3. They are not the same. The tank started with 4 litres, not 0. Go to M3.4.5.
   - **If you chose C:** Going up by the same amount each time is not the same as direct proportion. Direct proportion needs litres ÷ minutes to stay the same. Go to M3.4.5.

3. **63 litres**

   14 ÷ 2 = 7 and 21 ÷ 3 = 7, so there are 7 litres a minute. 9 × 7 = 63 litres.

   - **If you chose A:** You added 9 to 21. But the paint adds 7 litres every minute, so 6 more minutes add far more than 9 litres. Go to M3.4.5.
   - **If you chose B:** That is the litres each minute. The question asks about 9 minutes, so multiply 7 by 9. Go to M3.4.5.

4. **10 km**

   The line gives 4 km for every hour. 2.5 hours is 2.5 × 4 = 10 km. It lies halfway between 8 km and 12 km.

   - **If you chose A:** That is the distance at 2 hours. 2.5 hours is half an hour further on. Go to M3.4.5.
   - **If you chose B:** You added 1 km for the half hour. But Leila walks 4 km each hour, so half an hour is 2 km. Go to M3.4.5.
   - **If you chose D:** That is the distance at 3 hours. After 2.5 hours she has not walked as far. Go to M3.4.5.

5. **Graph A**

   Graph A is a straight line through the origin. Graph B is a straight line too. But it starts at 2, so its second amount divided by its first is not always the same.

   - **If you chose B:** Graph B is a straight line, but it starts at 2, above the origin. For direct proportion the line must start at 0 and 0. Go to M3.4.5.
   - **If you chose C:** A straight line is not enough. It must also start at the origin, where both amounts are 0. Go to M3.4.5.

6. **45 litres**

   20 ÷ 4 = 5 and 30 ÷ 6 = 5, so there are 5 litres a minute. 9 × 5 = 45 litres.

7. **21**

   12 ÷ 4 = 3, so the second is always 3 times the first. 7 × 3 = 21.

8. **15 carrots**

   1 person needs 24 ÷ 8 = 3 carrots. 5 people need 5 × 3 = 15 carrots.

M3.4.6. Rates and speed

1. **90 km per hour**

   360 ÷ 4 = 90. The train goes 90 km in each hour.

   - **If you chose A:** You multiplied 360 by 4. Speed is distance divided by time, not multiplied. Go to M3.4.6.
   - **If you chose C:** You took 4 away from 360. Speed is the distance for 1 hour, so divide 360 by the 4 hours. Go to M3.4.6.

2. **135 km**

   The bus goes 45 km in 1 hour. In 3 hours it goes 3 × 45 = 135 km.

   - **If you chose A:** You added the speed and the time. The bus goes 45 km in each of 3 hours, so multiply. Go to M3.4.6.
   - **If you chose B:** You divided 45 by 3. To find a distance, multiply the speed by the time. Go to M3.4.6.

3. **4 hours**

   Each hour is 15 km. 60 ÷ 15 = 4, so it takes 4 hours.

   - **If you chose A:** You multiplied 15 by 60. Time is distance divided by speed. Ask how many lots of 15 km are in 60 km. Go to M3.4.6.
   - **If you chose B:** You took 15 away from 60. Ask how many lots of 15 km fit into 60 km, so divide. Go to M3.4.6.

4. **Bag P**

   Bag P gives 9 ÷ 3 = 3 kg for each coin. Bag Q gives 14 ÷ 7 = 2 kg for each coin. 3 is more than 2.

   - **If you chose A:** Bag Q holds more rice, but it also takes more coins. Divide the rice by the coins for each bag. Go to M3.4.6.
   - **If you chose C:** Check each bag: 9 ÷ 3 and 14 ÷ 7. Are the answers the same? Go to M3.4.6.

5. **Tariq, at 5 km per hour. Priya goes 4 km per hour.**

   Priya: 12 ÷ 3 = 4 km per hour. Tariq: 10 ÷ 2 = 5 km per hour. 5 is more than 4, so Tariq is faster.

   - **If you chose A:** Priya goes further, but she takes 3 hours. Compare the distance for each hour, not the whole distance. Go to M3.4.6.
   - **If you chose C:** Check each speed: 12 ÷ 3 and 10 ÷ 2. Are they the same? Go to M3.4.6.

6. **15 km per hour**

   60 ÷ 4 = 15. The cyclist goes 15 km in each hour.

7. **5 hours**

   Each hour is 4 km. 20 ÷ 4 = 5, so it takes 5 hours.

8. **24**

   15 ÷ 5 = 3, so the second is always 3 times the first. 8 × 3 = 24.

M3.4.7. Maps and scale drawings

1. **3.5 km**

   7 × 50 000 = 350 000 cm. 350 000 ÷ 100 = 3 500 m. 3 500 ÷ 1 000 = 3.5 km.

   - **If you chose A:** You read 350 000 as metres. The scale is in centimetres. Change 350 000 cm to metres, then to kilometres. Go to M3.4.7.
   - **If you chose C:** You divided by 1 000 only. Divide by 100 first to get 3 500 m. Then divide by 1 000. Go to M3.4.7.

2. **1.5 km**

   1 cm is 25 000 cm, which is 250 m. 6 cm is 6 × 250 = 1 500 m. That is 1.5 km.

   - **If you chose A:** You read 150 000 as metres. The scale is in centimetres. 150 000 cm is 1 500 m. Go to M3.4.7.
   - **If you chose B:** You divided by 1 000 only. First divide by 100 to change centimetres to metres, then by 1 000 for kilometres. Go to M3.4.7.

3. **1.6 cm**

   800 m = 80 000 cm. Divide by the scale: 80 000 ÷ 50 000 = 1.6 cm.

   - **If you chose B:** That is ten times too long. 800 m is 80 000 cm, and 80 000 ÷ 50 000 = 1.6. Check your zeros. Go to M3.4.7.
   - **If you chose C:** You multiplied. Going from the ground to the map, divide by the scale. Go to M3.4.7.

4. **6 cm by 4 cm**

   1 cm stands for 2 m. 12 ÷ 2 = 6 cm and 8 ÷ 2 = 4 cm.

   - **If you chose A:** That uses 1 cm for each 1 m. Here 1 cm stands for 2 m, so divide each length by 2. Go to M3.4.7.
   - **If you chose C:** You multiplied by 2. A plan is smaller than the room, so divide by 2. Go to M3.4.7.

5. **1 : 100 000**

   1 km = 1 000 m = 100 000 cm. So 1 cm on the map stands for 100 000 cm on the ground.

   - **If you chose A:** 1 000 cm is only 10 m. 1 km is 1 000 m, which is much more in centimetres. Go to M3.4.7.
   - **If you chose C:** 1 000 000 cm is 10 km, ten times too much. 1 km is 100 000 cm. Go to M3.4.7.

6. **4.5 km**

   9 × 50 000 = 450 000 cm. That is 4 500 m, which is 4.5 km.

7. **7 cm by 5 cm**

   1 cm stands for 1 m. So 7 m is 7 cm and 5 m is 5 cm.

8. **12 km per hour**

   36 ÷ 3 = 12. The bike goes 12 km in each hour.

M3.4.8. Inverse proportion

1. **15 days**

   6 people for 10 days is 6 × 10 = 60 person-days. 4 people share 60 person-days: 60 ÷ 4 = 15 days.

   - **If you chose A:** You took away 2 because 4 is 2 fewer than 6. But fewer people need more days, not fewer. Go to M3.4.8.
   - **If you chose B:** You multiplied 4 by 10. But 6 × 10 = 60 person-days is the whole job, so divide 60 by 4. Go to M3.4.8.

2. **2 hours**

   The trip is 60 × 3 = 180 km. At 90 km per hour it takes 180 ÷ 90 = 2 hours.

   - **If you chose A:** A faster speed should take less time, not more. You scaled the time up by the same factor as the speed. Go to M3.4.8.
   - **If you chose B:** You halved the time, but 90 is not double 60. First find the distance: 60 × 3 = 180 km. Go to M3.4.8.

3. **The number of people sharing a cake and the size of each slice**

   More people sharing the same cake means a smaller slice for each. People × size of each slice is the whole cake, which stays the same.

   - **If you chose A:** If every notebook weighs the same, the total weight is the number of notebooks times one weight. That is direct proportion, not inverse. Go to M3.4.5.
   - **If you chose C:** The candle gets shorter as the hours go up, but hours × height does not stay the same. Inverse proportion needs the two amounts to multiply to the same number. Go to M3.4.8.

4. **8 people**

   The job is 1 × 24 = 24 person-days. For 3 days you need 24 ÷ 3 = 8 people.

   - **If you chose A:** You counted on by 2 from 4. But the days go down 24, 12, 6, 3, so the people go up in a different way. Use people × days = 24. Go to M3.4.8.
   - **If you chose C:** You copied a number from the days column. The people go with the days, so use people × days = 24. Go to M3.4.8.

5. **3 days**

   2 × 12 = 24 person-days. 8 people share it: 24 ÷ 8 = 3 days.

   - **If you chose A:** You multiplied 12 by 4 because 8 is 4 times 2. More people finish sooner, so the days must go down, not up. Go to M3.4.8.
   - **If you chose B:** You took 6 away from 12 because 8 is 6 more than 2. But the days go down by dividing, not by taking away. Go to M3.4.8.
   - **If you chose C:** You multiplied 8 by 12. That is the work of 8 people for 12 days, not the days they need. Go to M3.4.8.

6. **6 days**

   4 × 9 = 36 person-days. 6 people share it: 36 ÷ 6 = 6 days.

7. **3 hours**

   5 × 6 = 30 pump-hours. 10 pumps share it: 30 ÷ 10 = 3 hours.

8. **4 km**

   8 × 50 000 = 400 000 cm. That is 4 000 m, which is 4 km.

About this book

For learners who can simplify a fraction such as 12/16 to 3/4 and know their times tables. In many schools this fits children of about 11 to 13, and older learners who never met ratio can start here too. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it. Each chapter also has a short page for the adult, with three questions to ask.

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