●●●○○ Level 3 · Mathematics M3.3

Fractions, decimals and percentages

Calculate with fractions, decimals and percentages, and change one into another.

Simplify and calculate with fractions, then do the same with decimals. Change fractions into decimals and back, and use percentages to find parts and compare amounts.

YOU ARE READY FOR THIS BOOK IF

You can show with a drawing that 2/4 is the same as 1/2.

One step up	M4.2 Percentages, proportion and growth
This book	**M3.3 Fractions, decimals and percentages**
One step down	M2.4 Fractions

Contents

M3.3.1 Equal fractions and simplest form

M3.3.2 Adding and taking away fractions

M3.3.3 Multiplying and dividing fractions

M3.3.4 Tenths, hundredths and thousandths

M3.3.5 Adding and taking away decimals

M3.3.6 Multiplying and dividing decimals

M3.3.7 Fractions and decimals

M3.3.8 Percentages

Key words, A to Z

Written and drawn by attnom. Free to read, print and share.

M3.3.1 CHAPTER 1

Equal fractions and simplest form

My answer is $^{12}/_{18}$ and the book says $^{2}/_{3}$. Who is right?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The book is right and I am wrong, because the numbers are different.
- ☐ **B** We are both right. The two fractions name the same amount.
- ☐ **C** I am right, because $^{12}/_{18}$ has bigger numbers.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Fractions, M2.4.6. Equal fractions on bars, such as $^{1}/_{2} = {}^{2}/_{4}$, and how to make one by doubling.
- Level 2, Fractions, M2.4.5. A fraction past 1, and writing $^{5}/_{4}$ as $1\ ^{1}/_{4}$.
- Level 3, Negative numbers, factors and powers, M3.1.6. How to find the highest common factor of two numbers.

BY THE END YOU CAN

- Simplify $^{12}/_{18}$ by dividing the top and bottom by 6
- Change $^{7}/_{3}$ into $2\ ^{1}/_{3}$ and back again

Two fractions can look different and still name the same amount. $^{1}/_{2}$ and $^{2}/_{4}$ are like that. We call them **equal fractions**.

The top number of a fraction is the **numerator**. It counts the parts you have. The bottom number is the **denominator**. It says how many equal parts make one whole.

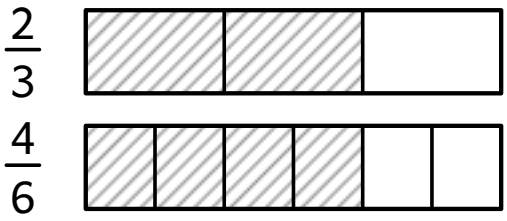
To make an equal fraction, multiply the top and the bottom by the same number. Every part is cut into smaller parts. The shaded amount does not change.

DRAW IT FIRST

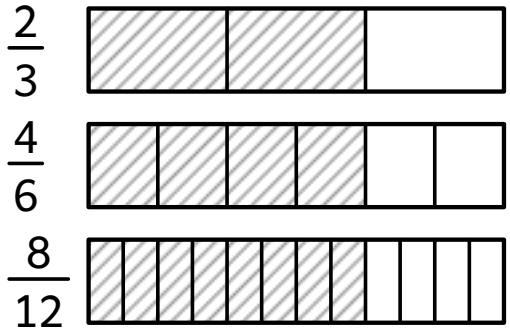
Cut every part into smaller parts



1 Draw a bar in 3 equal parts. Shade 2 parts. That is $^{2}/_{3}$.



2 Cut every part in 2. Now there are 6 parts and 4 are shaded. That is $^{4}/_{6}$.



3 Cut every part of the first bar in 4 instead. There are 12 parts and 8 are shaded. That is $^{8}/_{12}$.

The bars have the same length, and the shaded parts end in the same place. So $^{2}/_{3} = {}^{4}/_{6} = {}^{8}/_{12}$. Each time the top and the bottom were multiplied by the same number.

WORKED EXAMPLE

Write $^{3}/_{5}$ as a fraction with 20 on the bottom.

1. The bottom changes from 5 to 20. 20 ÷ 5 = 4, so multiply by 4.
2. Multiply the top by 4 too. 3 × 4 = 12.
3. The new bottom is 5 × 4 = 20. So $^{3}/_{5} = {}^{12}/_{20}$.

Answer: $^{3}/_{5} = {}^{12}/_{20}$

You can go the other way too. Divide the top and the bottom by the same number. This works only when both numbers are in that number's times table.

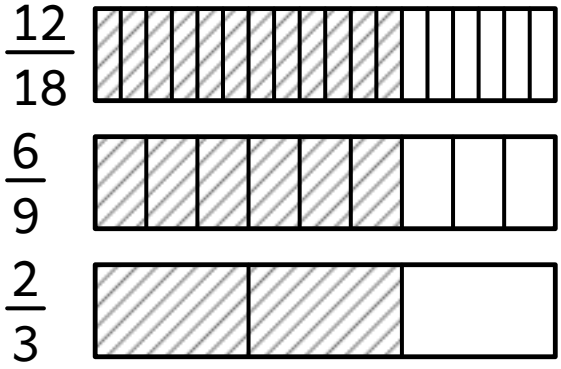
A fraction is in **simplest form** when only 1 divides both its top and its bottom. To reach it, divide both by their highest common factor (HCF).

WORKED EXAMPLE

Write $^{12}/_{18}$ in simplest form.

1. Find the highest common factor of 12 and 18. They share 2 and 3, so the HCF is 2 × 3 = 6.
2. Divide the top by 6. 12 ÷ 6 = 2.
3. Divide the bottom by 6. 18 ÷ 6 = 3.
4. So $^{12}/_{18} = {}^{2}/_{3}$. Only 1 divides both 2 and 3, so this is the simplest form.

Answer: $^{12}/_{18} = {}^{2}/_{3}$



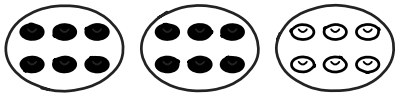
$^{12}/_{18} = {}^{6}/_{9} = {}^{2}/_{3}$. You can divide in small steps. Dividing by the HCF does it in one step.

SEE IT THREE WAYS

With things

Hana has 18 beans. 12 of them are red. She puts the beans in groups of 6. There are 3 groups, and 2 of them are red.

As a picture



red red not red

In symbols

$^{12}/_{18} = {}^{2}/_{3}$. Divide the top and the bottom by 6.

A fraction with a top bigger than its bottom is an **improper fraction**. $^{7}/_{3}$ is one. It is more than 1 whole. A **mixed number** has whole numbers and a fraction, like 2 $^{1}/_{3}$. Both write the same amount.

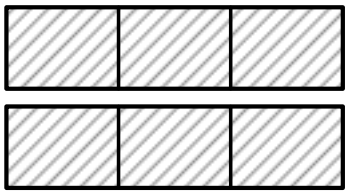
DRAW IT FIRST

Change $^{7}/_{3}$ into *a* mixed number



1 whole

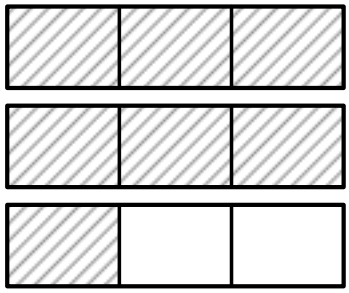
1 Shade *a* whole bar. It is cut in 3 equal parts, so it holds 3 thirds.



1 whole

1 whole

2 Shade *a* second whole bar. That is 3 more thirds, 6 thirds in *a*ll.



1 whole

1 whole

$\frac{1}{3}$

3 Shade 1 part of *a* third bar. That makes 7 thirds: 2 wholes and $^{1}/_{3}$.

WORKED EXAMPLE

Change $^{17}/_{5}$ into *a* mixed number.

1. Count how many fives fit in 17. 5 × 3 = 15, so 3 whole bars are full.
2. Take away what the full bars use. 17 − 15 = 2.
3. The 2 left over are fifths. They make the fraction $^{2}/_{5}$.
4. So $^{17}/_{5} = 3\ ^{2}/_{5}$.

Answer: $3\ ^{2}/_{5}$

To go back, multiply the whole number by the bottom number. Then add the top number. For $3\ ^{2}/_{5}$: 3 × 5 = 15, and 15 + 2 = 17. The bottom stays 5. So $3\ ^{2}/_{5} = {}^{17}/_{5}$.

TRY IT

Two strips, one amount

You need: two strips of paper of the same length, a pencil.

1. Fold the first strip into 3 equal parts. Open it and shade 2 parts.
2. Fold the second strip into 3 parts, then fold each part in half. Open it. It has 6 parts.
3. Shade 4 parts of the second strip.
4. Put the strips side by side, with the left ends together.

What you will see: Both shaded parts end at the same place. So $^{2}/_{3}$ and $^{4}/_{6}$ are the same amount. You multiplied the top and the bottom by 2.

If you cannot try it, this is what happens.



two thirds



four sixths

A WRONG IDEA MANY PEOPLE HAVE

~~To make an equal fraction, add the same number to the top and the bottom.~~

Why it feels right: Doing the same thing to both numbers looks fair.

The test that breaks it: Add 1 to the top and the bottom of $^1/_2$. You get $^2/_3$. Shade a bar for each. $^1/_2$ is 1 part of 2. $^2/_3$ is 2 parts of 3. The shaded parts do not end in the same place.

What is true: Multiply or divide the top and the bottom by the same number. Adding changes the amount.

KEY WORDS

equal fractions
fractions that name the same amount, like $^1/_2$ and $^2/_4$.

numerator
the top number of a fraction. It counts the parts you have.

denominator
the bottom number of a fraction. It says how many equal parts make one whole.

simplest form
a fraction whose top and bottom have no common factor except 1.

improper fraction
a fraction whose top is bigger than its bottom, like $^7/_3$.

mixed number
a whole number and a fraction together, like $2\ ^1/_3$.

BACK TO YOUR GUESS

The answer is B: We are both right. The two fractions name the same amount.

$^{12}/_{18}$ and $^2/_3$ are the same amount. The book gives $^2/_3$ because it is in simplest form. The top and the bottom are as small as they can be.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which of these is NOT equal to $^{2}/_{3}$?

   A $^{8}/_{10}$

   B $^{4}/_{6}$

   C $^{6}/_{9}$

   Answer at the back of the book.

2. What fraction of this bar is shaded? Choose the answer in simplest form.



8 equal parts

   A $^{1}/_{4}$

   B $^{6}/_{8}$

   C $^{3}/_{4}$

   D $^{3}/_{8}$

   Answer at the back of the book.

3. Which mixed number is the same as $^{11}/_{4}$?

   A $3\ ^{1}/_{4}$

   B $2\ ^{3}/_{4}$

   C $2\ ^{1}/_{4}$

   Answer at the back of the book.

4. Which improper fraction is the same as $2\ ^{3}/_{5}$?

   A $^{13}/_{5}$

   B $^{23}/_{5}$

   C $^{6}/_{5}$

   Answer at the back of the book.

5. How do you change $^{2}/_{3}$ into twelfths?

   A Multiply the bottom by 4 and leave the top

   B Add 9 to the top and the bottom

   C Multiply the top and the bottom by 4

   Answer at the back of the book.

6. Write a fraction equal to $^{3}/_{4}$ with 12 on the bottom.

   Answer at the back of the book.

7. Write $^{18}/_{24}$ in simplest form.

   Answer at the back of the book.

8. Change $^{23}/_{6}$ into a mixed number.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Equal fractions name the same amount. Multiply or divide the top and the bottom by the same number. Simplest form: divide by the HCF.

Draw: Two bars of the same length. Cut one in 3 and shade 2. Cut the other in 6 and shade 4. Write $^{2}/_{3} = {}^{4}/_{6}$.

M3.3.2 CHAPTER 2

Adding and taking away fractions

How can I add a third and a quarter when the parts are different sizes?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** $^1/_3 + {}^1/_4 = {}^7/_{12}$. Cut both into twelfths, then add.
- ☐ **B** $^1/_3 + {}^1/_4 = {}^2/_7$. Add the tops, and add the bottoms.
- ☐ **C** $^1/_3 + {}^1/_4 = {}^2/_{12}$. Add the tops, and keep the bottom 12.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Fractions, M2.4.8. Adding and taking away fractions that already have the same bottom number.
- Level 3, Negative numbers, factors and powers, M3.1.6. How to find the lowest common multiple of two numbers.

BY THE END YOU CAN

- Add $^1/_3 + {}^1/_4$ by finding a common bottom number
- Take away mixed numbers, like $2\ {}^1/_2 - 1\ {}^3/_4$

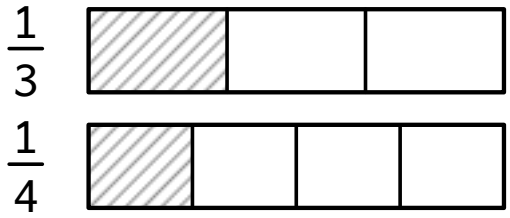
Fractions with the same bottom number are quick to add. $^2/_7 + {}^3/_7 = {}^5/_7$. The parts are the same size, so you add the tops and keep the bottom.

Parts of different sizes cannot be counted together, like 3 stones and 4 beans. First change both fractions so they have the same bottom number. That bottom number is a **common denominator**.

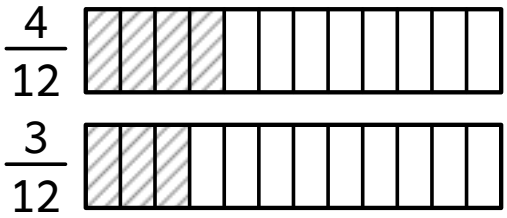
The best common denominator is the lowest common multiple (LCM) of the two bottom numbers. The LCM of 3 and 4 is 12. Both 3 and 4 go into 12 exactly.

DRAW IT FIRST

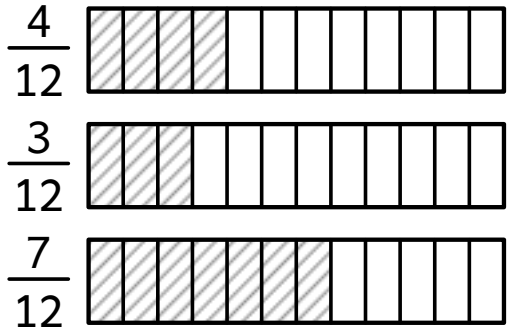
Add $^1/_3$ and $^1/_4$



1 Draw two bars of the same length. Cut one in 3 and shade 1 part. Cut the other in 4 and shade 1 part. The parts are not the same size.



2 Cut both bars into 12 parts. Now $^1/_3$ is $^4/_{12}$ and $^1/_4$ is $^3/_{12}$. The parts are the same size.



3 Count the shaded twelfths together. 4 + 3 = 7. So $^1/_3 + {}^1/_4 = {}^7/_{12}$.

WORKED EXAMPLE

Add $^{1}/_{3} + {}^{1}/_{4}$.

1. The bottom numbers are 3 and 4. The lowest common multiple of 3 and 4 is 12.
2. Change $^{1}/_{3}$. 12 ÷ 3 = 4, so $^{1}/_{3} = {}^{4}/_{12}$.
3. Change $^{1}/_{4}$. 12 ÷ 4 = 3, so $^{1}/_{4} = {}^{3}/_{12}$.
4. The bottom stays 12. Add the tops: 4 + 3 = 7.
5. So $^{1}/_{3} + {}^{1}/_{4} = {}^{7}/_{12}$.

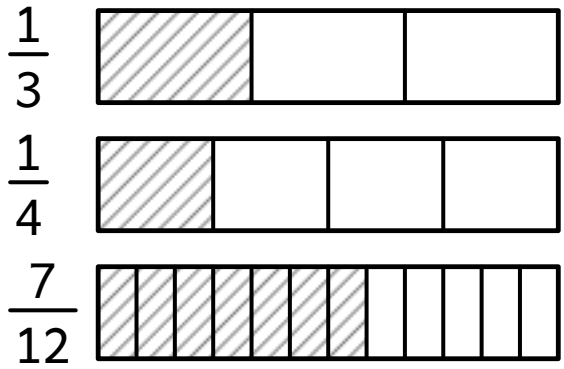
Answer: $^{7}/_{12}$

WORKED EXAMPLE

Add $^{3}/_{4} + {}^{5}/_{6}$.

1. The lowest common multiple of 4 and 6 is 12.
2. Change $^{3}/_{4}$. 3 × 3 = 9 and 4 × 3 = 12, so $^{3}/_{4} = {}^{9}/_{12}$.
3. Change $^{5}/_{6}$. 5 × 2 = 10 and 6 × 2 = 12, so $^{5}/_{6} = {}^{10}/_{12}$.
4. Add the tops. The bottom stays 12. 9 + 10 = 19.
5. The sum is $^{19}/_{12}$, which is more than 1. 19 − 12 = 7, so the answer is 1 $^{7}/_{12}$.

Answer: 1 $^{7}/_{12}$



$^{1}/_{3} + {}^{1}/_{4} = {}^{7}/_{12}$. It is a little more than half, and less than 1.

Taking away works the same way. Change both fractions to the same bottom number. Then take away the tops.

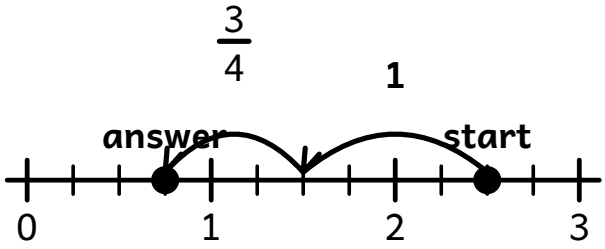
With mixed numbers, take away the fraction parts first. Sometimes you cannot, because the top is too small. Then change one whole into parts, as in this example.

WORKED EXAMPLE

Take away. $2\ ^{1}/_{2} - 1\ ^{3}/_{4}$

1. Change to a common bottom number. The lowest common multiple of 2 and 4 is 4.
2. $^{1}/_{2} = {}^{2}/_{4}$, so $2\ ^{1}/_{2} = 2\ ^{2}/_{4}$.
3. You cannot take 3 quarters from 2 quarters. Change 1 whole into 4 quarters. Now 2 quarters + 4 quarters = 6 quarters.
4. So $2\ ^{2}/_{4}$ is $1\ ^{6}/_{4}$. Take away: 1 − 1 = 0 wholes, and 6 − 3 = 3 quarters.
5. So $2\ ^{1}/_{2} - 1\ ^{3}/_{4} = {}^{3}/_{4}$.

Answer: $^{3}/_{4}$



Start at $2\ ^{1}/_{2}$. Hop left 1, then hop left $^{3}/_{4}$. You land on $^{3}/_{4}$.

SEE IT THREE WAYS

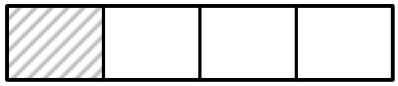
With things

Ravi eats $1/3$ of a flat bread. Lucía eats $1/4$ of a bread of the same size. How much bread did they eat in all?

As a picture



one third



one quarter



seven twelfths

In symbols

$$1/3 + 1/4 = 4/12 + 3/12 = 7/12.$$

TRY IT

Add on squared paper

You need: squared paper, a pencil.

1. Draw a bar 12 squares long and 1 square high.
2. Shade $1/3$ of the bar. That is 4 squares.
3. Shade $1/4$ of the bar next to it. That is 3 squares.
4. Count all the shaded squares.

What you will see: 4 + 3 = 7 squares are shaded out of 12. So $1/3 + 1/4 = 7/12$. Twelve squares let you cut the bar into both thirds and quarters.

If you cannot try it, this is what happens.



7 of 12 squares

A WRONG IDEA MANY PEOPLE HAVE

~~**$^1/_3 + {}^1/_4 = {}^2/_7$, because 1 + 1 = 2 and 3 + 4 = 7.**~~

Why it feels right: You add the tops and you add the bottoms. It looks like adding.

The test that breaks it: $^2/_7$ is smaller than $^1/_3$. Adding a quarter to a third cannot give less than the third you began with.

What is true: The bottom number is the size of the parts, so it is not added. Change both fractions to the same bottom number. Then add only the tops.

KEY WORDS

common denominator
a bottom number that two fractions can both be changed to.

lowest common multiple
the smallest number that is in both times tables. It is the best common denominator.

BACK TO YOUR GUESS

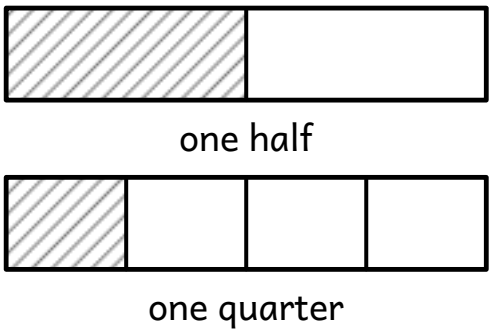
The answer is A: $^1/_3 + {}^1/_4 = {}^7/_{12}$. Cut both into twelfths, then add.

Parts of different sizes cannot be counted together. Cut both fractions into twelfths first. $^1/_3 = {}^4/_{12}$ and $^1/_4 = {}^3/_{12}$. Then 4 + 3 = 7, so the answer is $^7/_{12}$.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The bars show one half and one quarter. How much is shaded in all?



A $^1/_8$

B $^2/_6$

C $^2/_4$

D $^3/_4$

Answer at the back of the book.

2. What is the lowest common bottom number for $^1/_6$ and $^1/_9$?

A 3

B 54

C 18

D 15

Answer at the back of the book.

3. What is $^2/_3 + {}^1/_6$?

A $^3/_9$

B $^5/_6$

C $^3/_6$

Answer at the back of the book.

4. Take away. $3\ ^{1}/_{2} - 1\ ^{3}/_{4}$

   A $1\ ^{3}/_{4}$

   B $2\ ^{1}/_{4}$

   C $2\ ^{3}/_{4}$

   Answer at the back of the book.

5. Joseph has a rope $^{3}/_{4}$ m long. He cuts off a piece $^{1}/_{3}$ m long. How long is the rope that is left?

   A $^{2}/_{12}$ m

   B $^{2}/_{1}$ m

   C $^{5}/_{12}$ m

   Answer at the back of the book.

6. Add $^{1}/_{2} + ^{1}/_{6}$. Write the answer in simplest form.

   Answer at the back of the book.

7. Take away. $^{3}/_{4} - ^{1}/_{6}$

   Answer at the back of the book.

8. Take away. $4\ ^{1}/_{4} - 2\ ^{1}/_{2}$. Write the answer in simplest form.

   Answer at the back of the book.

9. **From an earlier chapter**

   Write $^{16}/_{24}$ in simplest form.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Parts must be the same size before you add. Find the LCM of the bottoms, change both fractions, then add the tops. The bottom stays.

Draw: Two bars of the same length, in thirds and quarters. Below them, a bar in twelfths with 7 shaded.

M3.3.3 CHAPTER 3

Multiplying and dividing fractions

What is two thirds of three quarters of a flat bread?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** $^5/_7$. Add the tops, and add the bottoms.
- ☐ **B** $^1/_2$. A part of three quarters must be less than three quarters.
- ☐ **C** More than $^3/_4$, because multiplying makes numbers bigger.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Fractions, M2.4.4. Finding a fraction of a number, such as $^2/_3$ of 12.
- Level 2, Times tables and sharing, M2.3.3. Dividing as finding how many groups of a size fit.

BY THE END YOU CAN

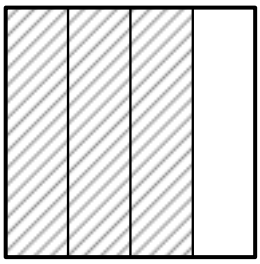
- Find $^2/_3$ of $^3/_4$ and write it as $^2/_3 \times {}^3/_4$
- Find how many quarters are in 3, and write $3 \div {}^1/_4$
- Divide one fraction by another, like $^3/_4 \div {}^1/_8$

Finding a fraction of an amount is the same as multiplying. $^1/_2$ of 12 is $^1/_2 \times 12 = 6$. The word "of" means times.

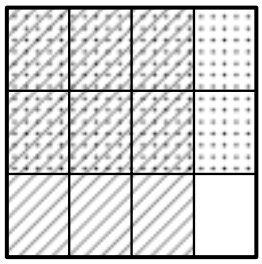
Now take a fraction of a fraction. We can see it on a square, cut in two ways at once.

DRAW IT FIRST

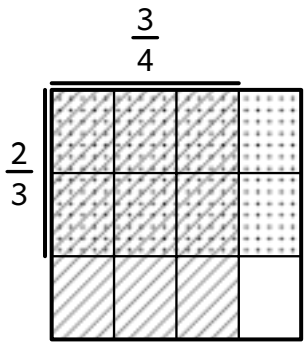
Find $^{2}/_{3}$ of $^{3}/_{4}$



1 Draw a square for 1 whole. Cut it in 4 columns. Shade 3 columns with lines. That is $^{3}/_{4}$ of the whole.



2 Cut the square in 3 rows. Shade 2 rows with dots. That is $^{2}/_{3}$ of the whole.



6 of 12 squares

3 The squares with lines and dots are $^{2}/_{3}$ of $^{3}/_{4}$. There are 6 of 12 squares, so $^{2}/_{3} \times {}^{3}/_{4} = {}^{6}/_{12}$.

WORKED EXAMPLE

Multiply $^{2}/_{3} \times {}^{3}/_{4}$.

1. Multiply the tops: 2 × 3 = 6.
2. Multiply the bottoms: 3 × 4 = 12.
3. So $^{2}/_{3} \times {}^{3}/_{4} = {}^{6}/_{12}$.
4. Simplify. The HCF of 6 and 12 is 6, so $^{6}/_{12} = {}^{1}/_{2}$.

Answer: $^{1}/_{2}$

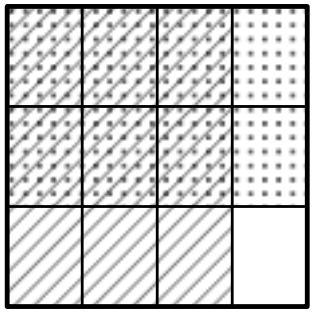
You can cancel before you multiply. In $^{2}/_{3} \times {}^{3}/_{4}$, a 3 is on the top and a 3 is on the bottom. Divide both by 3. You get $^{2}/_{4}$, which is $^{1}/_{2}$. Smaller numbers are quicker to work with.

SEE IT THREE WAYS

With things

Take a flat bread. Use $^{3}/_{4}$ of it. Then use $^{2}/_{3}$ of the part you took.

As a picture



6 of 12

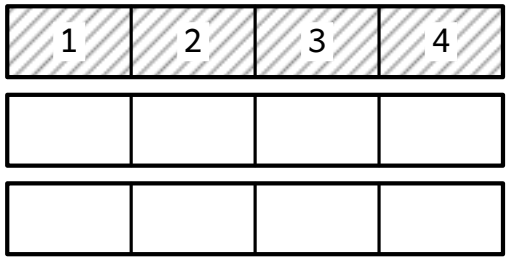
In symbols

$^{2}/_{3} \times {}^{3}/_{4} = {}^{6}/_{12} = {}^{1}/_{2}$. Multiply the tops, multiply the bottoms, then simplify.

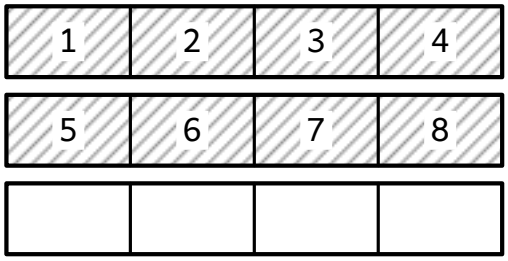
Now dividing. $3 \div {}^{1}/_{4}$ asks: how many quarters are in 3? We can count them.

DRAW IT FIRST

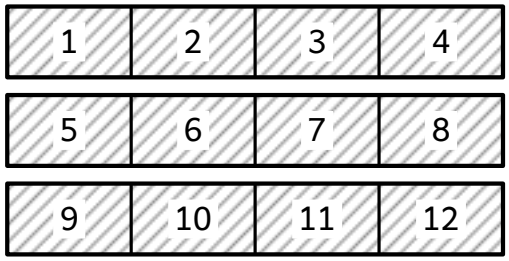
How many quarters are in 3?



1 Draw 3 wholes and cut each into quarters. Count the quarters in the first whole: 4.



2 Count on through the second whole. Now you have 8 quarters.



3 Count on through the third whole. There are 12 quarters in 3, so $3 \div {}^{1}/_{4} = 12$.

Each whole holds 4 quarters, so 3 wholes hold 3 × 4. Dividing by ${}^{1}/_{4}$ is the same as multiplying by 4. The number 4, written ${}^{4}/_{1}$, is the **reciprocal** of ${}^{1}/_{4}$. You flip a fraction to find its reciprocal.

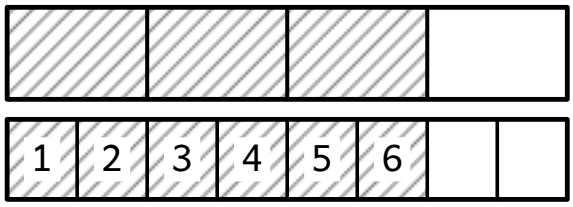
WORKED EXAMPLE

How many quarters are in 3? Work out $3 \div {}^{1}/_{4}$.

1. One whole is the same as 4 quarters.
2. So 3 wholes are 3×4 quarters. $3 \times 4 = 12$.
3. So $3 \div {}^{1}/_{4} = 12$.

Answer: 12

Now divide one fraction by another. How many eighths are in ${}^{3}/_{4}$? The two bars below have the same length. Count the eighths under the shaded quarters.



6 eighths

${}^{3}/_{4}$ is the same length as 6 eighths.

There are 6 eighths in ${}^{3}/_{4}$, so ${}^{3}/_{4} \div {}^{1}/_{8} = 6$. Dividing by a fraction is the same as multiplying by its reciprocal. ${}^{3}/_{4} \times 8 = 6$ too.

WORKED EXAMPLE

Divide ${}^{3}/_{4} \div {}^{1}/_{8}$.

1. Turn the divide into a times. The reciprocal of ${}^{1}/_{8}$ is 8, because 8 eighths make one whole.
2. So ${}^{3}/_{4} \div {}^{1}/_{8} = {}^{3}/_{4} \times 8$.
3. The bottom stays 4. Multiply the tops: $3 \times 8 = 24$.
4. So ${}^{3}/_{4} \times 8 = {}^{24}/_{4}$. Simplify: $24 \div 4 = 6$.
5. There are 6 eighths in ${}^{3}/_{4}$.

Answer: 6

TRY IT

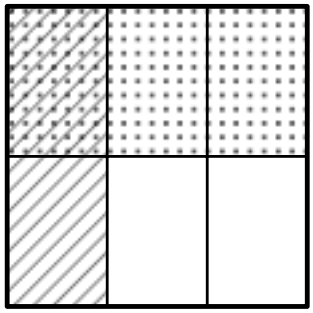
Fold *a* half of *a* third

You need: *a* sheet of paper, *a* pencil.

1. Fold the sheet into 3 equal parts, like *a* letter. Open it. Shade one part with lines.
2. Fold the sheet in half the other way. Open it.
3. Shade the top half of the whole sheet with dots.
4. Count the parts the folds make. Count the part with lines and dots.

What you will see: The folds make 6 equal parts, and 1 has lines and dots. So $^1/_2$ of $^1/_3$ is $^1/_6$. That is $^1/_2 \times {}^1/_3 = {}^1/_6$.

If you cannot try it, this is what happens.



1 of 6

A WRONG IDEA MANY PEOPLE HAVE

~~Dividing always makes *a* number smaller.~~

Why it feels right: With whole numbers bigger than 1 it is true. 12 ÷ 3 = 4, and 4 is less than 12.

The test that breaks it: Count the quarters in 3. There are 12. 12 is more than 3.

What is true: For *a* positive number, dividing by *a* fraction between 0 and 1 makes the answer bigger. You count smaller parts. Multiplying by such *a* fraction makes it smaller.

KEY WORDS

reciprocal

a fraction turned upside down. The reciprocal of $^1/_4$ is 4. Dividing by *a* fraction is multiplying by its reciprocal.

BACK TO YOUR GUESS

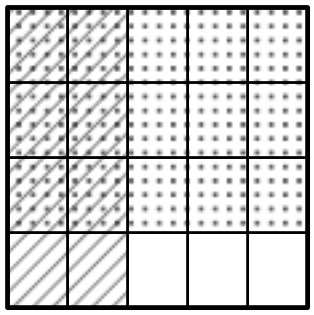
The answer is B: $^1/_2$. A part of three quarters must be less than three quarters.

Two thirds of three quarters is $^1/_2$. Taking *a* part of *an* amount gives less than the *amount*. Multiplying *a* positive number by *a* fraction less than 1 *makes* it smaller.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The columns with lines are $^2/_5$ of the square. The rows with dots are $^3/_4$ of the square. What fraction of the whole square has both lines and dots?



5 columns, 4 rows

A $^3/_{10}$

B $^6/_{20}$

C $^5/_9$

Answer at the back of the book.

2. What is $^{2}/_{3} \times {}^{3}/_{5}$?

   A $^{6}/_{8}$

   B $^{6}/_{15}$

   C $^{5}/_{8}$

   D $^{2}/_{5}$

   Answer at the back of the book.

3. How many quarters are in 3?

   A $^{3}/_{4}$

   B 7

   C 12

   Answer at the back of the book.

4. Which of these gives an answer bigger than 3?

   A $3 \times {}^{1}/_{2}$

   B $3 \div {}^{1}/_{2}$

   C $3 \div 1$

   Answer at the back of the book.

5. What is $^{3}/_{4} \div {}^{1}/_{8}$?

   A 6

   B $^{3}/_{32}$

   C $^{6}/_{8}$

   Answer at the back of the book.

6. Multiply. $^{3}/_{5} \times {}^{5}/_{6}$. Write the answer in simplest form.

   Answer at the back of the book.

7. How many $^{1}/_{5}$s are in 4?

   Answer at the back of the book.

8. Divide. $^{2}/_{3} \div {}^{1}/_{6}$

   Answer at the back of the book.

9. **From an earlier chapter**

   Add. $^{1}/_{4} + {}^{2}/_{3}$

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Of means times. Multiply the tops and the bottoms, then simplify. To divide by a fraction, multiply by its reciprocal.

Draw: A square cut in 4 columns and 3 rows. Shade 3 columns with lines and 2 rows with dots. Write $^{2}/_{3} \times {}^{3}/_{4} = {}^{1}/_{2}$.

M3.3.4 CHAPTER 4

Tenths, hundredths and thousandths

Is 0.45 bigger than 0.5, because 45 is bigger than 5?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. 45 is bigger than 5.
- ☐ **B** They are equal, because both have a 5 at the end.
- ☐ **C** No. 0.5 is bigger. It is 50 hundredths, and 0.45 is 45 hundredths.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.3. What a tenth is, and how 1.5 is written.
- Level 2, Big numbers and place value, M2.1.1. How the place of a digit tells what it is worth.

BY THE END YOU CAN

- Say what each digit in 3.472 is worth
- Read 3.47 and 3,47 as the same number
- Order decimals like 0.5, 0.45 and 0.405

In a whole number, each place is ten times bigger than the place to its right. Decimals carry on the same pattern to the right. Each place is ten times smaller than the one before it.

The **decimal point** separates the whole numbers from the parts. The first place after it holds **tenths**. The next holds **hundredths**, and the next holds **thousandths**.

Here is 3.472 in a place value chart. The 3 is in the ones place, so it is worth 3. The 4 is in the tenths place, just after the point. It is worth 4 tenths.

ones		tenths	hundredths	thousandths
3	.	4	7	2
3		0.4	0.07	0.002

The 7 is in the hundredths place and the 2 is in the thousandths place. Each digit has its own worth.

WORKED EXAMPLE

Say what each digit in 3.472 is worth.

1. The 3 is in the ones place, so it is worth 3.
2. The 4 is in the tenths place. 4 tenths is 4 ÷ 10 = 0.4.
3. The 7 is in the hundredths place. 7 hundredths is 7 ÷ 100 = 0.07.
4. The 2 is in the thousandths place. 2 thousandths is 2 ÷ 1 000 = 0.002.
5. So 3.472 = 3 + 0.4 + 0.07 + 0.002.

Answer: 3, 0.4, 0.07 and 0.002

A 0 holds a place. In 3.05 there are no tenths, so the tenths place has a 0. Never leave it out. 3.5 and 3.05 are different numbers.

In some places people write a comma where others write a point. They do the same job. 3,47 and 3.47 are the same number. Where a point is used, a comma can instead split a big number into groups, as in 1,250. This book uses a point, and writes 1 250 with a small space.

WORKED EXAMPLE

Write 3,47 with a decimal point, and say what its digits are worth.

1. A comma and a point both separate the whole numbers from the parts.
2. Swap the comma for a point. 3,47 becomes 3.47.
3. The 4 is in the tenths place. It is worth 4 ÷ 10 = 0.4.
4. The 7 is in the hundredths place. It is worth 7 ÷ 100 = 0.07.
5. So 3,47 and 3.47 are the same number, 3 + 0.4 + 0.07.

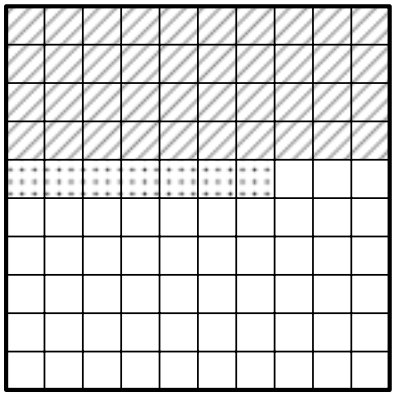
Answer: 3.47

SEE IT THREE WAYS

With things

Take a square made of 100 small squares. Shade 4 whole rows of 10, and then 7 more squares.

As a picture

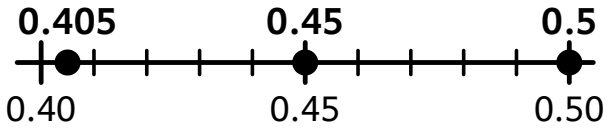


47 squares

In symbols

47 hundredths = 4 tenths + 7 hundredths = 0.47.

To compare decimals, compare the whole numbers first. If they are equal, the tenths decide. If the tenths are equal, look at the hundredths, and so on. It helps to write every number with the same number of places.



The number line shows the order: 0.405, then 0.45, then 0.5.

ones		tenths	hundredths	thousandths
0	.	5	0	0
0	.	4	5	0
0	.	4	0	5

The dashed zeros do not change a number. They let you compare place by place: 5 tenths is more than 4 tenths.

WORKED EXAMPLE

Put 0.5, 0.45 and 0.405 in order, smallest first.

1. Give all three the same number of places. 0.5 = 0.500 and 0.45 = 0.450.
2. Now read them as thousandths. 0.500 is 500 thousandths, 0.450 is 450, and 0.405 is 405.
3. Order the whole numbers. 405 is less than 450, and 450 is less than 500.
4. So the order is 0.405, 0.45, 0.5.

Answer: 0.405, 0.45, 0.5

TRY IT

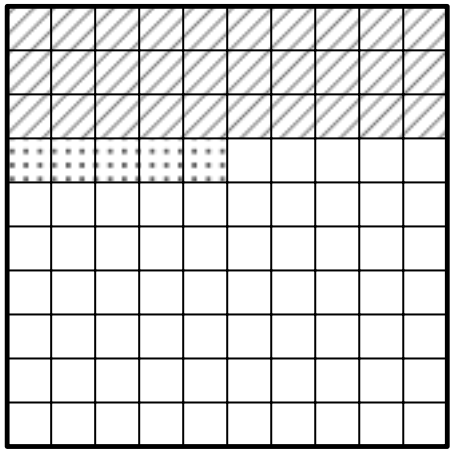
Shade a decimal on squared paper

You need: squared paper, a pencil.

1. Draw a square 10 squares wide and 10 squares high. It has 100 small squares.
2. Shade 3 whole rows of 10 with lines.
3. Shade 5 more squares in the next row with dots.
4. Say how many tenths and how many hundredths you shaded.

What you will see: You shaded 3 tenths and 5 hundredths. That is 35 hundredths, or 0.35.

If you cannot try it, this is what happens.



35 squares

A WRONG IDEA MANY PEOPLE HAVE

~~0.45 is bigger than 0.5, because 45 is bigger than 5.~~

Why it feels right: With whole numbers, a number with more digits is bigger. 45 is bigger than 5.

The test that breaks it: Write both with two places. 0.5 = 0.50. Now compare 50 hundredths with 45 hundredths. Or find them on a number line. 0.5 is to the right of 0.45.

What is true: Compare place by place from the left. 0.5 and 0.45 both have 0 ones, so look at the tenths. 0.5 has 5 tenths and 0.45 has only 4. More digits after the point does not mean a bigger number.

KEY WORDS

decimal point
the dot that separates the whole numbers from the parts, as in 3.47.

tenths
the first place after the decimal point. One tenth is $^1/_{10}$.

hundredths
the second place after the point. One hundredth is $^1/_{100}$.

thousandths
the third place after the point. One thousandth is $^1/_{1000}$.

BACK TO YOUR GUESS

The answer is C: No. 0.5 is bigger. It is 50 hundredths, and 0.45 is 45 hundredths.

0.5 is 5 tenths, which is 50 hundredths. 0.45 is only 45 hundredths. More digits after the point does not mean a bigger number.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. In 3.472, what is the 7 worth?

   A 0.007

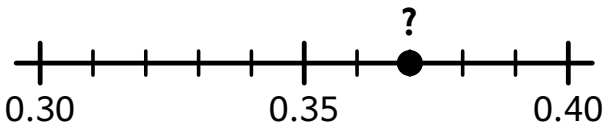
   B 7

   C 0.7

   D 0.07

   Answer at the back of the book.

2. What decimal is the dot on this number line?



   A 0.7

   B 0.37

   C 0.307

   Answer at the back of the book.

3. A scale in a shop shows 12,35 kg. Which number is that?

   A 12.35

   B 1235

   C 12.035

   Answer at the back of the book.

4. Which is the biggest?

   A 0.399

   B 0.35

   C 0.4

   Answer at the back of the book.

5. Which list is in order, smallest first?

   A 0.7, 0.07, 0.707

   B 0.07, 0.7, 0.707

   C 0.707, 0.7, 0.07

   Answer at the back of the book.

6. In 5.073, what is the 7 worth? Write it as a decimal.

   Answer at the back of the book.

7. Put these in order, smallest first. 0.6, 0.58, 0.508

   Answer at the back of the book.

8. A label reads 0,48. Write the number with a decimal point.

   Answer at the back of the book.

9. **From an earlier chapter**

   Multiply. $^{1}/_{2} \times {}^{3}/_{5}$

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Each place to the right of the point is ten times smaller. Tenths, hundredths, thousandths. Compare place by place from the left. 3,47 is 3.47.

Draw: A chart with the headings ones, tenths, hundredths and thousandths. Write 3.472 in it.

M3.3.5 CHAPTER 5

Adding and taking away decimals

What is 2.5 + 0.75, and why must the decimal points line up?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 2.80. Add 2 + 0 = 2, and 5 + 75 = 80 after the point.
- ☐ **B** 1.00. Add 25 and 75, then put the point back.
- ☐ **C** 3.25. Line up the points, so tenths are added to tenths.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Adding and taking away big numbers, M2.2.3. Adding in columns, and carrying from one column to the next.
- Level 2, Adding and taking away big numbers, M2.2.4. Taking away in columns, and borrowing from the next column.
- Level 2, Big numbers and place value, M2.1.5. Rounding a whole number to the nearest 10 or 100.

BY THE END YOU CAN

- Add and take away decimals by lining up the decimal points
- Multiply and divide decimals by 10, 100 and 1 000
- Round a decimal to the nearest whole or one decimal place

You add decimals the way you add whole numbers, in columns. Each digit must sit in a column with digits of the same worth. That is why the decimal points go in a line.

DRAW IT FIRST

Add 2.5 and 0.75 in a chart

ones		tenths	hundredths
2	.	5	
0	.	7	5

1 Write 2.5 and 0.75 with the decimal points in a line. Tenths sit under tenths.

ones		tenths	hundredths
2	.	5	0
0	.	7	5

2 Fill the empty place with a 0, so both numbers have two decimal places. 2.5 = 2.50.

	ones		tenths	hundredths
	2	.	5	0
+	0	.	7	5
	3	.	2	5

3 Add one column at a time, from the right. The sum is 3.25.

WORKED EXAMPLE

Add 2.5 + 0.75.

1. Line up the points. Write 2.5 as 2.50.
2. Add the hundredths: 0 + 5 = 5.
3. Add the tenths: 5 + 7 = 12, which is 1 one and 2 tenths.
4. Add the ones: 2 + 0 + 1 = 3.
5. So 2.5 + 0.75 = 3.25.

Answer: 3.25

Taking away works the same way. Line up the points and fill empty places with 0. If a digit is too small to take from, borrow 1 from the column on its left. That 1 is worth 10 in the column you are working in.

WORKED EXAMPLE

Take away. 5.2 − 1.75

1. Line up the points. Write 5.2 as 5.20.
2. Hundredths: 0 − 5 cannot be done. Borrow 1 tenth, which is 10 hundredths. 10 − 5 = 5.
3. Tenths: 1 tenth is left after borrowing. 1 − 7 cannot be done. Borrow 1 one, which is 10 tenths. 11 − 7 = 4.
4. Ones: 4 are left after borrowing. 4 − 1 = 3.
5. So 5.2 − 1.75 = 3.45.

Answer: 3.45

	ones		tenths	hundredths
	5	.	2	0
−	1	.	7	5
	3	.	4	5

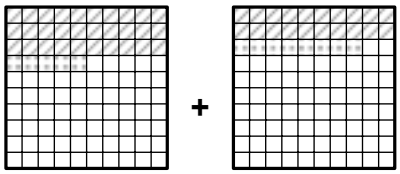
Top row: 5.20. Second row: take away 1.75. Under the double line: the answer, 3.45.

SEE IT THREE WAYS

With things

Mei shades 35 squares of a hundred grid. Kofi shades 28 squares of another one. How much is shaded in all?

As a picture



In symbols

0.35 + 0.28 = 0.63.
35 + 28 = 63 hundredths.

Now multiplying by 10. Every digit moves one place to the left, so it becomes 10 times bigger. The decimal point stays where it is. The digits move past it.

tens	ones		tenths	hundredths
	3	.	4	7
3	4	.	7	

3.47 × 10 = 34.7. Every digit moves one place to the left.

Multiplying by 100 moves the digits 2 places left. Multiplying by 1 000 moves them 3 places left. Dividing goes the other way. Dividing by 10, 100 or 1 000 moves the digits 1, 2 or 3 places to the right.

WORKED EXAMPLE

Work out 0.047 × 1 000.

1. 1 000 has 3 zeros, so move every digit 3 places to the left.
2. Do it one place at a time. First × 10 once: 0.047 × 10 = 0.47.
3. × 10 again: 0.47 × 10 = 4.7.
4. × 10 a third time: 4.7 × 10 = 47.
5. So 0.047 × 1 000 = 47.

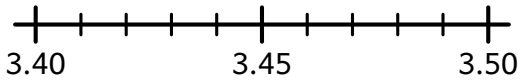
Answer: 47

The digits after the decimal point are the **decimal places**. 3.47 has 2 decimal places. To **round** a decimal means to change it to a nearby number with fewer decimal places. Look at the digit just after the place you keep.

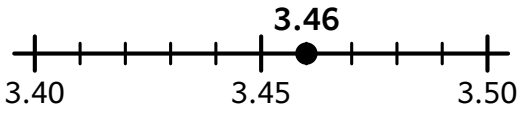
If that digit is 5 or more, round up. If it is 4 or less, leave the kept digit as it is. To round to a whole number, look at the tenths digit.

DRAW IT FIRST

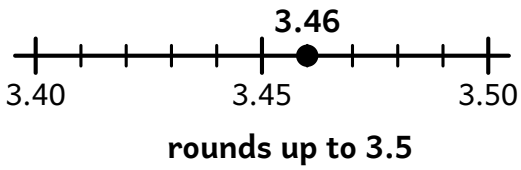
Round 3.46 to one decimal place



1 Draw the line from 3.40 to 3.50. The halfway mark is 3.45.



2 Mark 3.46. It is one step after the halfway mark.



rounds up to 3.5

3 3.46 is past halfway, so it is nearer to 3.5 than to 3.4. Round up: 3.46 ≈ 3.5.

TRY IT

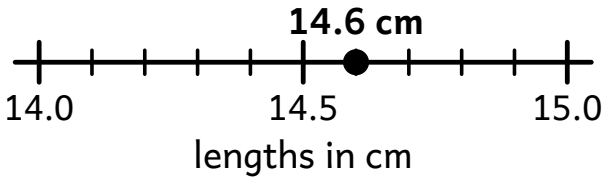
Round a length you measure

You need: a ruler marked in millimetres, a pencil or a stick.

1. Measure the length of the pencil in centimetres. Read the millimetre marks as tenths of a centimetre, such as 14.6 cm.
2. Look at the tenths digit. Is it 5 or more?
3. If it is, round up to the next whole centimetre. If not, keep the whole number of centimetres.
4. Check on the ruler. Is the end of the pencil nearer to the mark above or the mark below?

What you will see: A length of 14.6 cm is nearer to 15 cm than to 14 cm. So it rounds to 15 cm. Any length from 14.5 cm up to just under 15 cm rounds to 15 cm.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~To multiply by 10, add a zero. So 2.5 × 10 = 2.50.~~

Why it feels right: With whole numbers it works. 25 × 10 = 250 has a 0 added.

The test that breaks it: 2.50 and 2.5 are the same number. But 2.5 × 10 must be 10 times bigger. Move each digit one place left and you get 25.

What is true: Multiplying by 10 moves every digit one place to the left. 2.5 × 10 = 25. A zero is added to a whole number only because it fills the empty ones place.

KEY WORDS

decimal places
the digits after the decimal point. 3.47 has 2 decimal places.

round
to change a number to a nearby one that is shorter, such as 3.46 to 3.5.

BACK TO YOUR GUESS

The answer is C: 3.25. Line up the points, so tenths are added to tenths.

The answer is 3.25. Tenths must be added to tenths, and hundredths to hundredths. Lining up the decimal points puts every digit under a digit of the same worth. 2.50 + 0.75 = 3.25.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is 3.4 + 0.27?

   A 3.67

   B 0.61

   C 3.31

   Answer at the back of the book.

2. What is 5.2 − 1.75?

   A 4.55

   B 3.45

   C 3.55

   Answer at the back of the book.

3. What is 6.4 × 100?

   A 6.400

   B 64

   C 640

   Answer at the back of the book.

4. What is 35 ÷ 1 000?

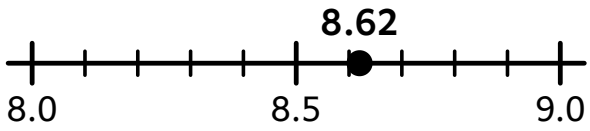
   A 0.035

   B 0.35

   C 0.0035

   Answer at the back of the book.

5. What is 8.62 rounded to the nearest whole number?



   A 8

   B 9

   C 8.6

   Answer at the back of the book.

6. What is 7.449 rounded to one decimal place?

   A 7.45

   B 7.5

   C 7.4

   Answer at the back of the book.

7. Add. 6.8 + 0.45

   Answer at the back of the book.

8. Work out 4.37 × 1 000.

   Answer at the back of the book.

9. **From an earlier chapter**

   In 5.206, what is the 2 worth? Write it as a decimal.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Line up the decimal points. Fill empty places with 0. × 10 moves digits left, ÷ 10 moves them right. To round, look at the next digit: 5 or more rounds up.

Draw: A chart with ones, tenths and hundredths. Write 2.50 above 0.75 with the points in a line, and add.

M3.3.6 CHAPTER 6

Multiplying and dividing decimals

Why is 0.3 × 0.2 equal to 0.06 and not 0.6?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 0.06. 3 × 2 = 6, and the two numbers have 2 decimal places in all.
- ☐ **B** 0.6. Multiply 3 × 2 = 6 and keep one decimal place.
- ☐ **C** 0.5. Multiplying by a decimal works like adding.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.8. Multiplying and dividing whole numbers with more than one digit.
- Level 2, Adding and taking away big numbers, M2.2.6. Using a rough answer to check a calculation.

BY THE END YOU CAN

- Multiply a decimal by a whole number, like 2.4 × 3
- Multiply two decimals, like 0.3 × 0.2
- Divide a decimal by a whole number, like 7.5 ÷ 3

Start with a decimal times a whole number. 2.4 × 3 means three groups of 2.4. You can add: 2.4 + 2.4 + 2.4.

DRAW IT FIRST

Work out 2.4 × 3 in *a* chart

ones		tenths
2	.	4

1 Write 2.4. It is 2 ones and 4 tenths.

ones		tenths
2	.	4
2	.	4
2	.	4

2 Write it 3 times, because × 3 means three groups of 2.4.

	ones		tenths
	2	.	4
+	2	.	4
+	2	.	4
	7	.	2

3 Add the columns. 3 × 4 tenths = 12 tenths, which is 1 one and 2 tenths. 3 × 2 ones = 6 ones, and 1 more makes 7 ones. The total is 7.2.

WORKED EXAMPLE

Work out 2.4 × 3.

1. Ignore the point for now. 24 × 3 = 72.
2. 2.4 has 1 decimal place, so the answer needs 1 decimal place.
3. Put the point back. 72 becomes 7.2.
4. Check with an estimate. 2.4 is about 2.5, and 2.5 × 3 = 7.5. That is close to 7.2.

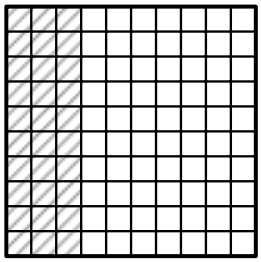
Answer: 7.2

An **estimate** is a quick answer that is close to the real one. Make one before you calculate. If your answer is far from your estimate, look for a mistake.

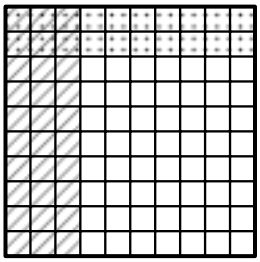
Now two decimals. 0.3 × 0.2 means 3 tenths of 2 tenths. A hundred grid shows it. The whole grid is 1. One column is 0.1.

DRAW IT FIRST

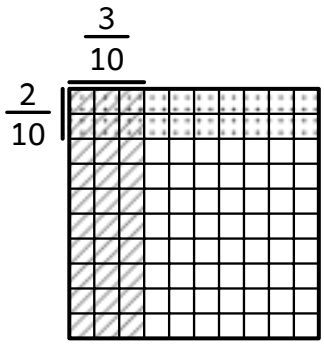
Find 0.3 of 0.2 on a hundred grid



1 Shade 3 columns of the grid with lines. That is 3 tenths, 0.3.



2 Shade 2 rows with dots. That is 2 tenths, 0.2.



6 of 100 squares

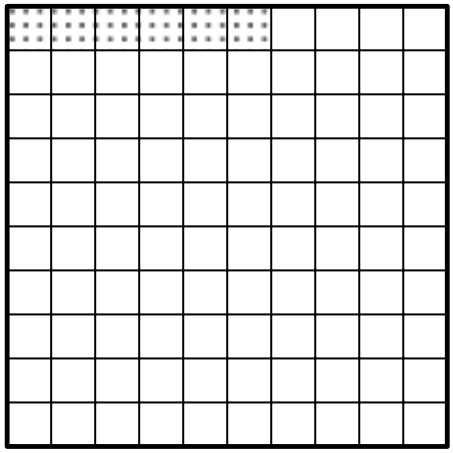
3 The squares with lines and dots are 6 of the 100 squares. That is 6 hundredths, 0.06.

WORKED EXAMPLE

Work out 0.3 × 0.2.

1. Ignore the points. 3 × 2 = 6.
2. Count the decimal places. 0.3 has 1 and 0.2 has 1. That is 2 in all.
3. The answer needs 2 decimal places. Write 6 with 2 places after the point: 0.06.
4. So 0.3 × 0.2 = 0.06.

Answer: 0.06



0.06 is 6 squares

The answer, 0.06, is 6 hundredths. It is less than 0.2, because it is only a part of 0.2.

Here is the rule. Multiply as if there were no points. Count the decimal places in both numbers. Write the whole number answer with that many decimal places. Then you may drop zeros at the end: 0.20 = 0.2.

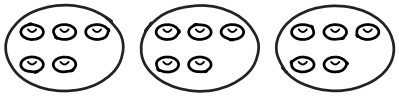
Now dividing a decimal by a whole number. Divide from the left, the biggest place first, as with whole numbers. When you reach the point, put a point in the answer in the same place. Then carry on with the tenths.

SEE IT THREE WAYS

With things

Ana shares 7.5 kg of rice between 3 bags. Each bean in the picture stands for 0.5 kg. She puts the same number of beans in each bag.

As a picture



In symbols

7.5 ÷ 3 = 2.5. Each bag gets 2.5 kg.

WORKED EXAMPLE

Work out 7.5 ÷ 3.

1. Divide the ones (there are no tens here). 7 ÷ 3 = 2, and 1 one is left over.
2. Write the 2 in the ones place of the answer. Put the decimal point after it.
3. The 1 left over is 10 tenths. Add the 5 tenths: 10 + 5 = 15 tenths.
4. Divide the tenths. 15 ÷ 3 = 5.
5. So 7.5 ÷ 3 = 2.5.

Answer: 2.5

TRY IT

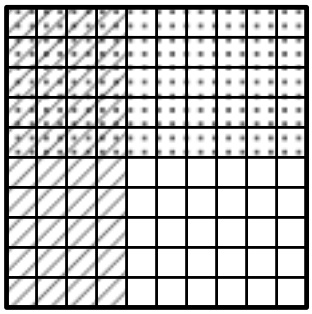
Find 0.4 of 0.5 on squared paper

You need: squared paper, a pencil, a ruler.

1. Draw a square 10 squares wide and 10 squares high. It is one whole.
2. Shade 4 columns with lines. That is 0.4.
3. Shade 5 rows with dots. That is 0.5.
4. Count the squares that have both lines and dots.

What you will see: There are 20 squares with both. 20 hundredths is 0.2. So 0.4 × 0.5 = 0.2.

If you cannot try it, this is what happens.



20 of 100 squares

A WRONG IDEA MANY PEOPLE HAVE

~~0.3 × 0.2 = 0.6, because 3 × 2 = 6.~~

Why it feels right: The digits are right. And you may think the answer must have one decimal place, like the numbers.

The test that breaks it: Find 0.3 of 0.2. That is a part of 0.2, so it must be less than 0.2. But 0.6 is more than 0.2. On the hundred grid you count only 6 squares of 100.

What is true: Write the whole number answer with as many decimal places as both numbers together. 1 + 1 = 2 places, so 6 becomes 0.06. Then you may drop zeros at the end: 0.20 = 0.2.

KEY WORDS

estimate
a quick *a*nswer that is close to the real one. You use it to check your work.

decimal places
the digits *a*fter the decimal point. 0.06 has 2 decimal places.

BACK TO YOUR GUESS

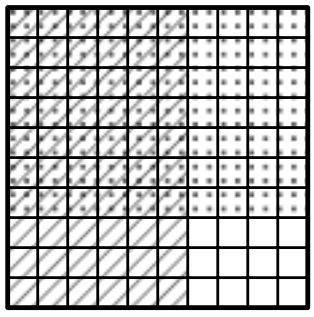
The answer is A: 0.06. 3 × 2 = 6, and the two numbers have 2 decimal places in all.

0.3 × 0.2 = 0.06. A tenth of *a* tenth is *a* hundredth. 3 tenths of 2 tenths is 6 hundredths. The answer must be smaller than 0.2, because it is only *a* part of 0.2.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The grid has 10 columns and 10 rows. The columns with lines are 0.6 of the whole. The rows with dots are 0.7 of the whole. Which sentence does the grid show?



10 columns, 10 rows

A 0.6 × 0.7 = 4.2

B 0.6 × 0.7 = 0.42

C 0.6 × 0.7 = 1.3

Answer at the back of the book.

2. What is 2.4 × 3?

   A 72

   B 6.12

   C 7.2

   Answer at the back of the book.

3. What is 0.4 × 0.6?

   A 0.24

   B 2.4

   C 0.024

   Answer at the back of the book.

4. What is 7.2 ÷ 3?

   A 24

   B 2.4

   C 0.24

   Answer at the back of the book.

5. Make an estimate first. Which answer to 4.1 × 2.9 is sensible?

   A 118.9

   B 1.189

   C 11.89

   Answer at the back of the book.

6. Work out 3.6 × 4.

   Answer at the back of the book.

7. Work out 0.7 × 0.4.

   Answer at the back of the book.

8. Work out 9.6 ÷ 4.

   Answer at the back of the book.

9. **From an earlier chapter**

Take away. 6.3 − 2.45

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Multiply as whole numbers, then count the decimal places in both numbers. The answer has that many. Make an estimate to check. When you divide, keep the point in line.

Draw: A hundred grid. Shade 3 columns with lines and 2 rows with dots. Write 0.3 × 0.2 = 0.06.

M3.3.7 CHAPTER 7

Fractions and decimals

A fraction is a division. So what decimal is $^3/_8$?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 0.375. Divide 3 by 8.
- ☐ **B** 0.38. Write the 3 and the 8 either side of the point.
- ☐ **C** 2.666... Divide 8 by 3.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.3. Sharing a number equally, which is what dividing means.
- Level 2, Fractions, M2.4.6. Equal fractions, which you need to simplify a fraction.

BY THE END YOU CAN

- Change $^3/_8$ into a decimal by dividing
- Change 0.35 into a fraction in simplest form
- Spot a decimal that repeats, like $^1/_3$ = 0.333...

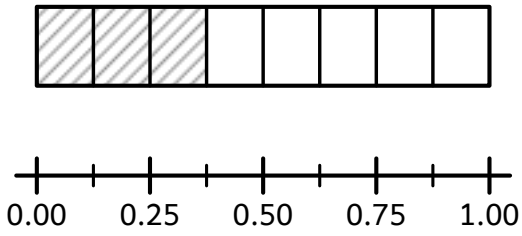
A fraction is a division in disguise. $^3/_8$ means 3 ÷ 8. To change a fraction into a decimal, divide the top number by the bottom number.

DRAW IT FIRST

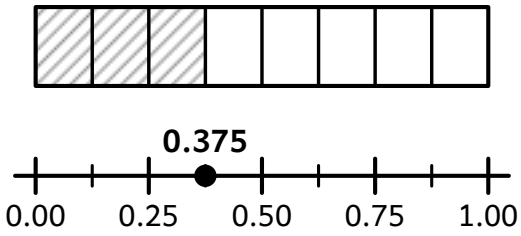
Find $^{3}/_{8}$ on *a* number line



1 The bar is one whole, cut into 8 equal parts. 3 parts are shaded. That is $^{3}/_{8}$.



2 Draw a number line under the bar, from 0 to 1. Mark a tick at every eighth. Each tick is 0.125 further on.



3 The shaded part ends at the third tick. That point is 0.375. So $^{3}/_{8}$ = 0.375.

WORKED EXAMPLE

Change $^{3}/_{8}$ into a decimal.

1. Work out 3 ÷ 8. Write 3 as 3.000, so there are tenths, hundredths and thousandths to divide.
2. Ones: 3 ÷ 8 = 0, with 3 left over. 3 ones are 30 tenths.
3. Tenths: 30 ÷ 8 = 3, with 6 left over.
4. Hundredths: 60 ÷ 8 = 7, with 4 left over.
5. Thousandths: 40 ÷ 8 = 5, with 0 left over.
6. The digits of the answer are 3, 7 and 5. So $^{3}/_{8}$ = 0.375.

Answer: 0.375

A decimal that ends, like 0.375, is a **terminating decimal**. Its division finishes with nothing left over.

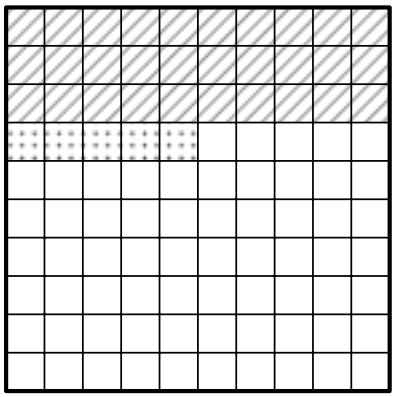
To go the other way, read the place value. 0.35 is 35 hundredths, so 0.35 = $^{35}/_{100}$. Then simplify the fraction.

SEE IT THREE WAYS

With things

Take a hundred grid. Shade 35 squares. The whole grid is 100 squares.

As a picture



35 of 100 squares

In symbols

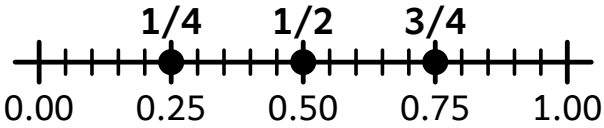
0.35 = $^{35}/_{100}$ = $^{7}/_{20}$. Divide the top and the bottom by 5.

WORKED EXAMPLE

Change 0.35 into a fraction in simplest form.

1. The last digit, 5, is in the hundredths place. So 0.35 is 35 hundredths.
2. Write it as a fraction: $^{35}/_{100}$.
3. Find the HCF of 35 and 100. They share one 5, so the HCF is 5.
4. Divide the top and the bottom by 5. 35 ÷ 5 = 7 and 100 ÷ 5 = 20.
5. So 0.35 = $^{7}/_{20}$.

Answer: $^{7}/_{20}$

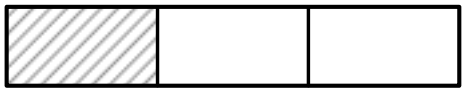


Three fractions worth knowing as decimals: $^{1}/_{4}$ = 0.25, $^{1}/_{2}$ = 0.5 and $^{3}/_{4}$ = 0.75.

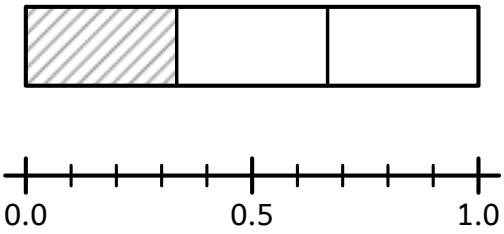
Some divisions never end. Try $^{1}/_{3}$. Divide 1 by 3 and watch what is left over each time.

DRAW IT FIRST

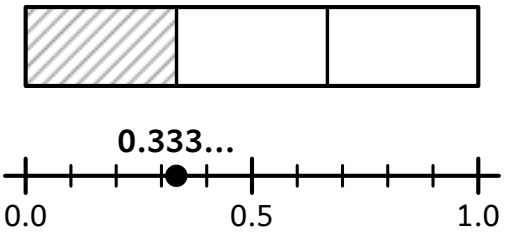
Find $^1/_3$ on *a* number line



1 The bar is one whole, cut into 3 equal parts. 1 part is shaded. That is $^1/_3$.



2 Draw *a* number line under the bar with *a* tick *a*t every tenth. The end of the shaded part is *a* little after the tick for 0.3.



3 The point is *a*t 0.333... The 3 goes on for ever.

WORKED EXAMPLE

Change $^1/_3$ into a decimal. Does the decimal stop?

1. Work out 1 ÷ 3. Write 1 as 1.000. Ones: 1 ÷ 3 = 0, with 1 left over. 1 one is 10 tenths.
2. Tenths: 10 ÷ 3 = 3, with 1 left over.
3. Hundredths: the 1 left over is 10 hundredths. 10 ÷ 3 = 3, with 1 left over.
4. Thousandths: it is the same sum again. 10 ÷ 3 = 3, with 1 left over.
5. The same 1 comes back every time, so the digit 3 comes back every time.
6. So $^1/_3$ = 0.333... and the 3s never stop.

Answer: No, it repeats: 0.333...

A decimal whose digits repeat for ever is a **recurring decimal**. We write three dots after the repeating digits. Some fractions give a recurring decimal and some give a terminating one. A fraction in simplest form ends when its bottom number is made only of 2s and 5s.

TRY IT

Check $^1/_4$ with a strip of 20 squares

You need: squared paper, a pencil.

1. Draw a strip 20 squares long and 1 square high. It is one whole.
2. Cut the strip into 4 equal parts. Each part is 5 squares.
3. Shade one part. That is $^1/_4$.
4. Write the shaded squares as a fraction of 20. Then change it into hundredths.

What you will see: You shaded 5 of 20 squares. $^5/_{20} = {}^{25}/_{100}$, which is 0.25. So $^1/_4 = 0.25$.

If you cannot try it, this is what happens.



5 of 20 squares

A WRONG IDEA MANY PEOPLE HAVE

~~$^3/_8$ = 0.38, because you write the 3 and the 8 on either side of the point.~~

Why it feels right: The fraction has two numbers and a decimal has two parts. It looks like a match.

The test that breaks it: Try the rule on $^1/_2$. It would give 0.12. But one half is 0.5. The rule fails.

What is true: A fraction is a division. $^3/_8$ means 3 ÷ 8, which is 0.375.

KEY WORDS

terminating decimal
a decimal that ends, like 0.375.

recurring decimal

a decimal whose digits repeat for ever, like 0.333...

BACK TO YOUR GUESS

The answer is A: 0.375. Divide 3 by 8.

$^3/_8$ means 3 ÷ 8, and 3 ÷ 8 = 0.375. The top number is divided by the bottom number. Writing the numbers either side of a point does not work: by that rule $^1/_2$ would be 0.12.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is $^3/_5$ as a decimal?

   A 0.35

   B 0.6

   C 1.666...

   Answer at the back of the book.

2. The bar is one whole. What decimal is the shaded part?



one whole

   A 0.34

   B 0.25

   C 0.75

   Answer at the back of the book.

3. Write 0.6 as a fraction in simplest form.

   A $^{3}/_{5}$

   B $^{6}/_{10}$

   C $^{1}/_{6}$

   Answer at the back of the book.

4. Write 0.125 as a fraction in simplest form.

   A $^{125}/_{1000}$

   B $^{1}/_{8}$

   C $^{1}/_{125}$

   Answer at the back of the book.

5. Which of these fractions gives a decimal that repeats for ever?

   A $^{3}/_{8}$

   B $^{1}/_{4}$

   C $^{1}/_{3}$

   Answer at the back of the book.

6. Change $^{5}/_{8}$ into a decimal.

   Answer at the back of the book.

7. Write 0.45 as a fraction in simplest form.

   Answer at the back of the book.

8. **From an earlier chapter**

   Work out 2.4 × 5.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A fraction is a division. $^{3}/_{8}$ means 3 ÷ 8 = 0.375. Read a decimal by its place value, then simplify. If the same remainder comes back, the decimal repeats.

Draw: A bar cut in 8 with 3 shaded. Under it a number line 0 to 1 with a point at 0.375.

M3.3.8 CHAPTER 8

Percentages

What does 35% mean, and what is 15% of 80?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 1 200. Multiply 15 × 80.
- ☐ **B** 12. 10% of 80 is 8, and 5% is half of that, 4.
- ☐ **C** 65. Take 15 away from 80.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Fractions, M2.4.4. Finding a fraction of a number, such as $^1/_4$ of 12.
- Level 2, Times tables and sharing, M2.3.8. Multiplying and dividing numbers by 10 and 100.

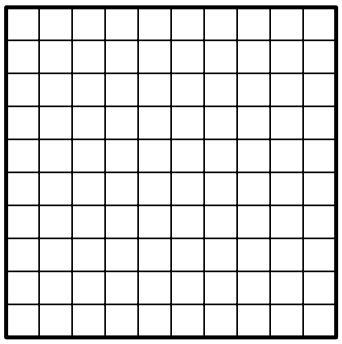
BY THE END YOU CAN

- Write 35% as a fraction and as a decimal
- Find 15% of 80 by finding 10% and 5%
- Write one amount as a percentage of another

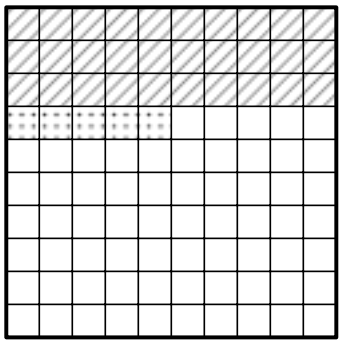
Percent means out of 100. The sign % stands for it. 35% means 35 out of every 100. As a fraction that is $^{35}/_{100}$. As a decimal it is 0.35.

DRAW IT FIRST

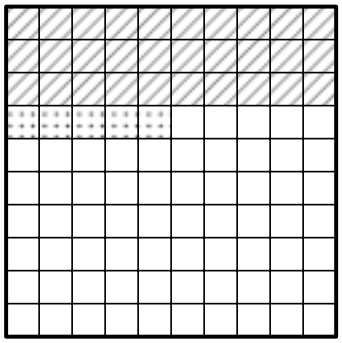
Show 35% on *a* hundred grid



1 A hundred grid is one whole. That is 100%. Each small square is 1%.



2 Shade 35 squares. That is 35%.



35% = 0.35

3 35 out of 100 is $^{35}/_{100}$, and 35 ÷ 100 = 0.35. So 35% = 0.35.

WORKED EXAMPLE

Write 35% as a fraction and as a decimal.

1. 35% means 35 out of 100. As a fraction that is $^{35}/_{100}$.
2. Divide the top and the bottom by 5. 35 ÷ 5 = 7 and 100 ÷ 5 = 20.
3. So 35% = $^{7}/_{20}$ in simplest form.
4. For the decimal, divide 35 by 100. 35 ÷ 100 = 0.35.
5. So 35% = $^{7}/_{20}$ = 0.35.

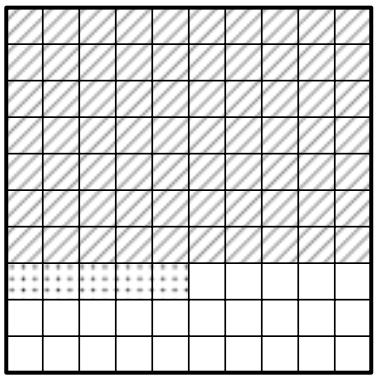
Answer: $^{7}/_{20}$ and 0.35

SEE IT THREE WAYS

With things

A tank holds 100 litres. Joseph pours in 75 litres. Each square in the picture stands for 1 litre.

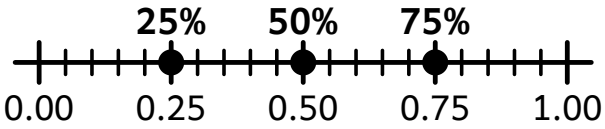
As a picture



75 litres of 100 litres

In symbols

75 litres out of 100 litres is 75% = $^{3}/_{4}$ = 0.75.

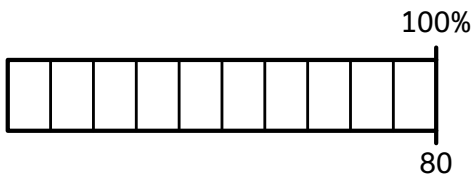


25% = 0.25 = $^{1}/_{4}$. 50% = 0.5 = $^{1}/_{2}$. 75% = 0.75 = $^{3}/_{4}$.

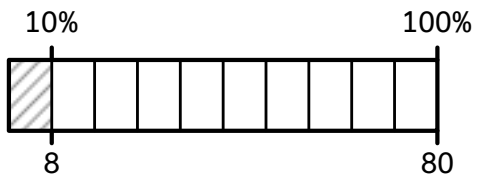
To find a percentage of an amount, build it from parts you know. 10% is a tenth, so divide by 10. 1% is a hundredth, so divide by 100. 5% is half of 10%.

DRAW IT FIRST

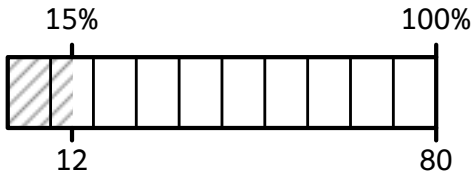
Find 15% of 80 kg



1 Draw a bar for the whole amount, 80 kg. It is 100%. Cut it into 10 equal sections of 10% each.



2 One section is 10%. 10% of 80 kg is 80 ÷ 10 = 8 kg.



3 5% is half of a section, which is 4 kg. So 15% is 8 kg + 4 kg = 12 kg.

WORKED EXAMPLE

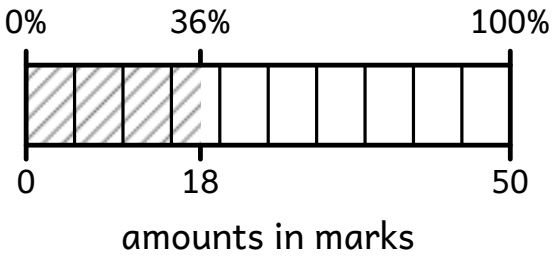
Find 15% of 80 kg.

1. Find 10% first. 10% is a tenth, so divide 80 by 10. 80 ÷ 10 = 8.
2. 5% is half of 10%. Half of 8 is 4.
3. Add them. 15% = 10% + 5% = 8 + 4 = 12.
4. So 15% of 80 kg is 12 kg.

Answer: 12 kg

Now the other way. To write one amount as a percentage of another, write it as a fraction first. Then change the fraction into hundredths. The bottom number must

become 100. When the bottom does not go into 100, divide the part by the whole. Then multiply by 100. For 12 out of 40, 12 ÷ 40 = 0.3, which is 30%.



18 marks out of 50 marks is 36% of the bar.

WORKED EXAMPLE

Mei scores 18 marks out of 50. Write 18 as a percentage of 50.

1. Write it as a fraction: $^{18}/_{50}$.
2. To get 100 on the bottom, multiply by 100 ÷ 50 = 2.
3. Multiply the top by 2 as well. 18 × 2 = 36.
4. Check by dividing. 18 ÷ 50 = 0.36.
5. So $^{18}/_{50} = {}^{36}/_{100}$, which is 36%.

Answer: 36%

TRY IT

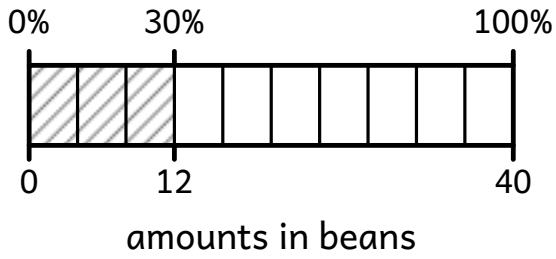
Find 30% of 40 beans

You need: 40 beans or small stones, a table to lay them on.

1. Count out 40 beans. That is 100%.
2. Share them into 10 equal heaps. Each heap is 10% of the beans.
3. Count the beans in one heap.
4. Take 3 heaps. Count the beans in them.

What you will see: Each heap has 4 beans. 3 heaps have 12 beans. So 30% of 40 beans is 12 beans.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~To find 5% of an amount, divide it by 5.~~

Why it feels right: The number in 5% is 5, and dividing by a number seems to find a part.

The test that breaks it: Find 5% of 80. Half of 10% is half of 8, which is 4. Now divide 80 by 5. You get 16, which is 20% of 80.

What is true: 5% is 5 out of 100, which is $^{1}/_{20}$. So divide by 20. 80 ÷ 20 = 4.

KEY WORDS

percent
out of 100. The sign % stands for it. 35% is 35 out of 100.

percentage

an amount written as a part of 100, such as 36% of the marks.

BACK TO YOUR GUESS

The answer is B: 12. 10% of 80 is 8, and 5% is half of that, 4.

15% of 80 is 12. Percent means out of 100. 10% of 80 is a tenth of it, 8. 5% is half of 10%, which is 4. And 8 + 4 = 12.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Write 60% as a fraction in simplest form.

   A $^{3}/_{5}$

   B $^{6}/_{10}$

   C $^{6}/_{100}$

   Answer at the back of the book.

2. Write 8% as a decimal.

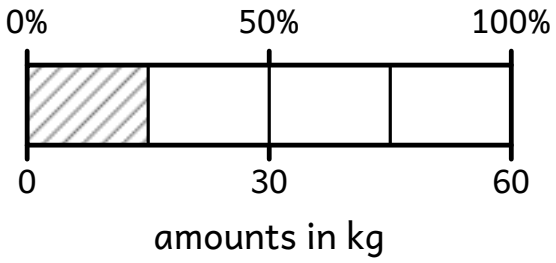
   A 0.008

   B 0.8

   C 0.08

   Answer at the back of the book.

3. The bar stands for an amount of 60 kg. How much is 25% of it?



A 25 kg

B 15 kg

C 35 kg

Answer at the back of the book.

4. What is 15% of 60?

A 9

B 4

C 75

Answer at the back of the book.

5. Write 12 out of 40 as a percentage.

A 3.333...%

B 12%

C 30%

Answer at the back of the book.

6. Find 15% of 120 kg.

Answer at the back of the book.

7. Write 21 out of 50 as a percentage.

Answer at the back of the book.

8. **From an earlier chapter**

Change $^{1}/_{8}$ into a decimal.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Percent means out of 100. 35% = $^{35}/_{100}$ = 0.35. To find 15%, find 10% by dividing by 10, then 5% as half of that, and add.

Draw: A hundred grid with 35 squares shaded. Write 35% = 0.35 under it.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

common denominator
a bottom number that two fractions can both be changed to. M3.3.2

decimal places
the digits after the decimal point. 3.47 has 2 decimal places. M3.3.5, M3.3.6

decimal point
the dot that separates the whole numbers from the parts, as in 3.47. M3.3.4

denominator
the bottom number of a fraction. It says how many equal parts make one whole. M3.3.1

equal fractions
fractions that name the same amount, like $^{1}/_{2}$ and $^{2}/_{4}$. M3.3.1

estimate
a quick answer that is close to the real one. You use it to check your work. M3.3.6

hundredths
the second place after the point. One hundredth is $^{1}/_{100}$. M3.3.4

improper fraction
a fraction whose top is bigger than its bottom, like $^{7}/_{3}$. M3.3.1

lowest common multiple
the smallest number that is in both times tables. It is the best common denominator. M3.3.2

mixed number
a whole number and a fraction together, like $2\ ^{1}/_{3}$. M3.3.1

numerator
the top number of a fraction. It counts the parts you have. M3.3.1

percent
out of 100. The sign % stands for it. 35% is 35 out of 100. M3.3.8

percentage
an amount written as a part of 100, such as 36% of the marks. M3.3.8

reciprocal

a fraction turned upside down. The reciprocal of $^1/_4$ is 4. Dividing by a fraction is multiplying by its reciprocal. M3.3.3

recurring decimal
a decimal whose digits repeat for ever, like 0.333... M3.3.7

round
to change a number to a nearby one that is shorter, such as 3.46 to 3.5. M3.3.5

simplest form
a fraction whose top and bottom have no common factor except 1. M3.3.1

tenths
the first place after the decimal point. One tenth is $^1/_{10}$. M3.3.4

terminating decimal
a decimal that ends, like 0.375. M3.3.7

thousandths
the third place after the point. One thousandth is $^1/_{1000}$. M3.3.4

Answers

M3.3.1. Equal fractions and simplest form

1. **$^{8}/_{10}$**

   To get from 3 to 10 you cannot multiply by a whole number. $^{8}/_{10} = {}^{4}/_{5}$, which is not $^{2}/_{3}$. The other two come from multiplying $^{2}/_{3}$ by 2 and by 3.

   - **If you chose B:** Multiply the top and the bottom of $^{2}/_{3}$ by 2. You get $^{4}/_{6}$, so it is equal to $^{2}/_{3}$. Go to M3.3.1.
   - **If you chose C:** Multiply the top and the bottom of $^{2}/_{3}$ by 3. You get $^{6}/_{9}$, so it is equal to $^{2}/_{3}$. Go to M3.3.1.

2. **$^{3}/_{4}$**

   6 of 8 parts are shaded, so the fraction is $^{6}/_{8}$. The HCF of 6 and 8 is 2. 6 ÷ 2 = 3 and 8 ÷ 2 = 4.

   - **If you chose A:** That is the part that is not shaded. 2 parts of 8 are not shaded, and $^{2}/_{8} = {}^{1}/_{4}$. Count the shaded parts. Go to M3.3.1.
   - **If you chose B:** That is the right amount, but it is not in simplest form. 6 and 8 are both in the 2 times table, so divide both by 2. Go to M3.3.1.
   - **If you chose D:** You divided the top by 2 but not the bottom. To keep the amount, divide both numbers by the same number. Go to M3.3.1.

3. **$2\ ^{3}/_{4}$**

   4 × 2 = 8, and 11 − 8 = 3. So there are 2 wholes and 3 quarters.

   - **If you chose A:** 3 whole bars need 3 × 4 = 12 quarters, and there are only 11. 2 wholes use 8 quarters, and 3 are left. Go to M3.3.1.
   - **If you chose C:** 2 wholes use 8 quarters. 11 − 8 = 3 quarters are left over, not 1. Go to M3.3.1.

4. **$^{13}/_{5}$**

   2 × 5 = 10, and 10 + 3 = 13. The bottom stays 5.

   - **If you chose B:** That puts the 2 and the 3 side by side. 2 wholes are 2 × 5 = 10 fifths. Then add the 3 fifths. Go to M3.3.1.
   - **If you chose C:** You multiplied 2 × 3. Multiply the whole number by the bottom number, 5. Then add the top number, 3. Go to M3.3.1.

5. **Multiply the top and the bottom by 4**

   12 ÷ 3 = 4, so multiply by 4. 2 × 4 = 8 and 3 × 4 = 12. So $^{2}/_{3} = {}^{8}/_{12}$.

   - **If you chose A:** That gives $^{2}/_{12}$, which is $^{1}/_{6}$. That is a much smaller amount. The top must change too. Go to M3.3.1.
   - **If you chose B:** Adding 9 gives $^{11}/_{12}$, which is not $^{2}/_{3}$. Multiply both numbers instead. Go to M3.3.1.

6. **$^{9}/_{12}$**

   12 ÷ 4 = 3, so multiply the top and the bottom by 3. 3 × 3 = 9 and 4 × 3 = 12.

7. **$^{3}/_{4}$**

   The HCF of 18 and 24 is 6. 18 ÷ 6 = 3 and 24 ÷ 6 = 4. Only 1 divides both 3 and 4.

8. **3 $^{5}/_{6}$**

   6 × 3 = 18, and 23 − 18 = 5. So there are 3 wholes and 5 sixths left over.

M3.3.2. Adding and taking away fractions

1. **$^{3}/_{4}$**

   One half is $^{2}/_{4}$. Then $^{2}/_{4} + {}^{1}/_{4} = {}^{3}/_{4}$.

   - **If you chose A:** You multiplied the fractions. $^{1}/_{2} \times {}^{1}/_{4} = {}^{1}/_{8}$, but here you are putting the parts together, so you add. Go to M3.3.2.
   - **If you chose B:** You added the tops and added the bottoms. The parts are different sizes, so that does not work. One half is 2 quarters. Go to M3.3.2.
   - **If you chose C:** You added the tops, 1 + 1, but kept the bottom 4 without changing $^{1}/_{2}$. One half is $^{2}/_{4}$, so add 2 + 1. Go to M3.3.2.

2. **18**

   Multiples of 6: 6, 12, 18. Multiples of 9: 9, 18. The lowest one in both lists is 18.

   - **If you chose A:** 3 is a factor of 6 and 9. A common bottom number must be a multiple of both. 6 and 9 must both go into it. Go to M3.3.2.
   - **If you chose B:** 54 is a common multiple of 6 and 9, so it works. But it is not the lowest one, and it gives bigger numbers to work with. Go to M3.3.2.
   - **If you chose D:** You added 6 + 9. 6 does not go into 15 exactly. Use the lowest common multiple instead. Go to M3.3.2.

3. **$^{5}/_{6}$**

   $^{2}/_{3} = {}^{4}/_{6}$. Then $^{4}/_{6} + {}^{1}/_{6} = {}^{5}/_{6}$.

   - **If you chose A:** You added the tops and added the bottoms. The parts are different sizes, so that does not work. Change $^{2}/_{3}$ into sixths first. Go to M3.3.2.
   - **If you chose C:** You changed the bottom to 6 but not the top. $^{2}/_{3}$ is $^{4}/_{6}$, not $^{2}/_{6}$. Go to M3.3.2.

4. **$1\ ^{3}/_{4}$**

   $3\ ^{1}/_{2} = 3\ ^{2}/_{4} = 2\ ^{6}/_{4}$. Take away $1\ ^{3}/_{4}$: 2 − 1 = 1 whole and 6 − 3 = 3 quarters.

   - **If you chose B:** You took the smaller quarters from the bigger ones. $^{1}/_{2}$ is $^{2}/_{4}$, and that is less than $^{3}/_{4}$. Change 1 whole into 4 quarters first. Go to M3.3.2.
   - **If you chose C:** You changed 1 whole into 4 quarters but did not take 1 from the wholes. $3\ ^{2}/_{4}$ becomes $2\ ^{6}/_{4}$. Go to M3.3.2.

5. **$^{5}/_{12}$ m**

   The LCM of 4 and 3 is 12. $^{3}/_{4} = {}^{9}/_{12}$ and $^{1}/_{3} = {}^{4}/_{12}$. 9 − 4 = 5, so $^{5}/_{12}$ m is left.

   - **If you chose A:** You changed the bottoms to 12 but did not change the tops. $^{3}/_{4} = {}^{9}/_{12}$ and $^{1}/_{3} = {}^{4}/_{12}$. Go to M3.3.2.
   - **If you chose B:** You took away the tops and took away the bottoms. That gives 2 m, which is more than the rope he started with. Go to M3.3.2.

6. **$^{2}/_{3}$**

   The LCM of 2 and 6 is 6. $^{1}/_{2} = {}^{3}/_{6}$. Then 3 + 1 = 4, so the sum is $^{4}/_{6}$. In simplest form that is $^{2}/_{3}$.

7. **$^{7}/_{12}$**

   The LCM of 4 and 6 is 12. $^{3}/_{4} = {}^{9}/_{12}$ and $^{1}/_{6} = {}^{2}/_{12}$. 9 − 2 = 7.

8. **$1\ ^{3}/_{4}$**

   $^{1}/_{2} = {}^{2}/_{4}$. You cannot take 2 quarters from 1 quarter, so change 1 whole into 4 quarters. $4\ ^{1}/_{4} = 3\ ^{5}/_{4}$. Then 3 − 2 = 1 whole and 5 − 2 = 3 quarters.

9. **$^{2}/_{3}$**

   The HCF of 16 and 24 is 8. 16 ÷ 8 = 2 and 24 ÷ 8 = 3.

M3.3.3. Multiplying and dividing fractions

1. **$^{3}/_{10}$**

   The square has 5 × 4 = 20 small squares. Squares with both lines and dots are 2 columns of 3 rows, which is 6. $^{6}/_{20} = {}^{3}/_{10}$.

   - **If you chose B:** That is the right amount, but it is not in simplest form. 6 and 20 are both in the 2 times table, so divide both by 2. Go to M3.3.3.
   - **If you chose C:** You added the tops and added the bottoms. A part of a part is found by multiplying, not adding. Go to M3.3.3.

2. **$^{2}/_{5}$**

   2 × 3 = 6 and 3 × 5 = 15. $^{6}/_{15}$ simplifies to $^{2}/_{5}$, because the HCF of 6 and 15 is 3.

   - **If you chose A:** You multiplied the tops, but you added the bottoms. Multiply the bottoms too: 3 × 5 = 15. Go to M3.3.3.
   - **If you chose B:** That is the right amount, but it is not in simplest form. 6 and 15 are both in the 3 times table, so divide both by 3. Go to M3.3.3.
   - **If you chose C:** You added the tops and added the bottoms. To multiply, multiply the tops and multiply the bottoms. Go to M3.3.3.

3. **12**

   One whole holds 4 quarters. 3 wholes hold 3 × 4 = 12 quarters.

   - **If you chose A:** That is 3 quarters of one whole. The question asks how many quarters fit into 3 whole ones, which is many more. Go to M3.3.3.
   - **If you chose B:** You added 3 + 4. Each whole holds 4 quarters, so 3 wholes hold 3 × 4 quarters. Go to M3.3.3.

4. **3 ÷ $^{1}/_{2}$**

   There are 6 halves in 3, so 3 ÷ $^{1}/_{2}$ = 6. The other two give 1 $^{1}/_{2}$ and 3.

   - **If you chose A:** Multiplying by a fraction less than 1 makes the number smaller. Half of 3 is 1 $^{1}/_{2}$. Go to M3.3.3.
   - **If you chose C:** Dividing by 1 leaves 3. It does not make it bigger. Go to M3.3.3.

5. **6**

   Multiply by the reciprocal. $^{3}/_{4} \times 8 = {}^{24}/_{4} = 6$. There are 6 eighths in $^{3}/_{4}$.

   - **If you chose B:** You multiplied the fractions. To divide, count how many eighths fit in $^{3}/_{4}$. Or multiply by the reciprocal of $^{1}/_{8}$, which is 8. Go to M3.3.3.
   - **If you chose C:** You changed $^{3}/_{4}$ into eighths, $^{6}/_{8}$, which is right so far. $^{6}/_{8}$ means 6 eighths. The question asks how many eighths there are, and that is 6. Go to M3.3.3.

6. **$^{1}/_{2}$**

   $3 \times 5 = 15$ and $5 \times 6 = 30$. $^{15}/_{30}$ simplifies to $^{1}/_{2}$, because the HCF of 15 and 30 is 15.

7. **20**

   One whole holds 5 fifths. 4 wholes hold $4 \times 5 = 20$ fifths.

8. **4**

   Multiply by the reciprocal. $^{2}/_{3} \times 6 = {}^{12}/_{3} = 4$.

9. **$^{11}/_{12}$**

   The LCM of 4 and 3 is 12. $^{1}/_{4} = {}^{3}/_{12}$ and $^{2}/_{3} = {}^{8}/_{12}$. $3 + 8 = 11$.

M3.3.4. Tenths, hundredths and thousandths

1. **0.07**

   The 7 is in the hundredths place. 7 hundredths is $7 \div 100 = 0.07$.

   - **If you chose A:** 0.007 is 7 thousandths. Thousandths are the third place after the point, and the 7 is in the second. Go to M3.3.4.
   - **If you chose B:** You read the 7 as 7 ones. The 7 is two places after the point, in the hundredths place. Go to M3.3.4.
   - **If you chose C:** 0.7 is 7 tenths. The 7 is in the second place after the point, which holds hundredths. Go to M3.3.4.

2. **0.37**

   Each small step is one hundredth. The dot is 7 steps after 0.30, so it is 0.30 + 0.07 = 0.37.

   - **If you chose A:** Each small step is one hundredth, not one tenth. The dot is 7 steps after 0.30, so it is 0.30 + 0.07. Go to M3.3.4.
   - **If you chose C:** 0.307 is 3 tenths and 7 thousandths. The dot is 7 hundredths after 0.30, so the 7 goes in the hundredths place. Go to M3.3.4.

3. **12.35**

   A comma and a point do the same job. Swap the comma for a point: 12,35 is 12.35.

   - **If you chose B:** You left out the comma. Here the comma does the job of the decimal point. It separates the whole numbers from the parts. Go to M3.3.4.
   - **If you chose C:** You added a 0 that is not there. 35 hundredths is written .35, with the 3 in the tenths place. Go to M3.3.4.

4. **0.4**

   All three have 0 ones, so compare the tenths first. 0.4 has 4 tenths, and the others have only 3. So 0.4 is the biggest.

   - **If you chose A:** It has more digits, but that does not make it bigger. 0.399 has 3 tenths and 0.4 has 4. Go to M3.3.4.
   - **If you chose B:** Count the tenths. 0.35 has 3 tenths, but 0.4 has 4 tenths. Go to M3.3.4.

5. **0.07, 0.7, 0.707**

   Write them with three places. 0.070, 0.700 and 0.707. 70 is less than 700, and 700 is less than 707.

   - **If you chose A:** 0.7 is 7 tenths, which is more than 0.07. Write both with three places: 0.700 and 0.070. Go to M3.3.4.
   - **If you chose C:** That list goes from biggest to smallest. The question asks for smallest first. Go to M3.3.4.

6. **0.07**

   The 0 is in the tenths place and the 7 is in the hundredths place. 7 hundredths is 7 ÷ 100 = 0.07.

7. **0.508, 0.58, 0.6**

   Write them with three places. 0.600, 0.580 and 0.508. 508 is less than 580, and 580 is less than 600.

8. **0.48**

   A comma and a point do the same job. Swap the comma for a point.

9. **$^{3}/_{10}$**

   $1 \times 3 = 3$ and $2 \times 5 = 10$. $^{3}/_{10}$ is already in simplest form.

M3.3.5. Adding and taking away decimals

1. **3.67**

   Write 3.4 as 3.40. Hundredths: 0 + 7 = 7. Tenths: 4 + 2 = 6. Ones: 3. The sum is 3.67.

   - **If you chose B:** You added 34 and 27 as whole numbers and ignored the points. The 4 is 4 tenths and the 27 is hundredths, so they are in different columns. Go to M3.3.5.
   - **If you chose C:** You added 4 tenths and 27 hundredths as if they were the same: 4 + 27 = 31. Write 3.4 as 3.40, then add each column. Go to M3.3.5.

2. **3.45**

   Write 5.2 as 5.20. Hundredths: 10 − 5 = 5. Tenths: 11 − 7 = 4. Ones: 4 − 1 = 3. The answer is 3.45.

   - **If you chose A:** You took the smaller digit from the bigger one in each column. 2 tenths cannot lose 7 tenths, so you must borrow. Go to M3.3.5.
   - **If you chose C:** After you borrow 1 tenth for the hundredths, only 1 tenth is left, not 2. So the tenths are 11 − 7 = 4. Go to M3.3.5.

3. **640**

   100 has 2 zeros, so move each digit 2 places left. The 6 moves to the hundreds, the 4 moves to the tens, and a 0 fills the ones.

   - **If you chose A:** Adding zeros after the last decimal digit does not change a number. 6.400 is still 6.4. Move the digits left. Go to M3.3.5.
   - **If you chose B:** That is 6.4 × 10. 100 has two zeros, so move the digits 2 places left. Go to M3.3.5.

4. **0.035**

   Move each digit 3 places to the right. 35 = 35.0, so the 5 moves to the thousandths and the 3 to the hundredths: 0.035.

   - **If you chose B:** That is 35 ÷ 100. 1 000 has three zeros, so move the digits 3 places right. Go to M3.3.5.
   - **If you chose C:** That moved the digits 4 places. Three zeros mean three places. Go to M3.3.5.

5. **9**

   The tenths digit is 6, which is 5 or more, so round up. On the line, 8.62 is nearer to 9 than to 8.

   - **If you chose A:** You cut off the decimals. Look at the tenths digit: 6 is 5 or more, so round up. Go to M3.3.5.
   - **If you chose C:** 8.6 is 8.62 rounded to one decimal place. The question asks for a whole number. Go to M3.3.5.

6. **7.4**

   Keep one decimal place: 7.4. The next digit is 4, which is 4 or less, so the 4 stays.

   - **If you chose A:** 7.45 has two decimal places. The question asks for one. Go to M3.3.5.
   - **If you chose B:** You looked at the 9 and rounded up. Only the digit right after the place you keep counts. That digit is 4, so keep the 4. Go to M3.3.5.

7. **7.25**

   Write 6.8 as 6.80, line up the points and add each column. Hundredths: 0 + 5 = 5. Tenths: 8 + 4 = 12, so write 2 and carry 1. Ones: 6 + 0 + 1 = 7.

8. **4 370**

   1 000 has 3 zeros, so move every digit 3 places left. The 4 moves to the thousands, and a 0 fills the ones place.

9. **0.2**

   The 2 is in the tenths place. 2 tenths is 2 ÷ 10 = 0.2.

M3.3.6. Multiplying and dividing decimals

1. **0.6 × 0.7 = 0.42**

   6 columns and 7 rows cross in 6 × 7 = 42 squares. 42 of 100 squares is 42 hundredths, which is 0.42.

   - **If you chose A:** The squares with lines and dots are 42 of the 100. Each square is one hundredth, so 42 hundredths is 0.42, not 4.2. Go to M3.3.6.
   - **If you chose C:** You added 0.6 and 0.7. The picture shows a part of a part, which is found by multiplying. Go to M3.3.6.

2. **7.2**

   24 × 3 = 72. 2.4 has 1 decimal place, so the answer is 7.2.

   - **If you chose A:** The digits are right, but you forgot to put the decimal point back. 2.4 has 1 decimal place, so the answer has 1. Go to M3.3.6.
   - **If you chose B:** You wrote 2 × 3 = 6 and 4 × 3 = 12 side by side. 12 tenths is 1 one and 2 tenths, so the 1 moves into the ones. 6 + 1 = 7. Go to M3.3.6.

3. **0.24**

   4 × 6 = 24. There are 2 decimal places in all, so the answer is 0.24.

   - **If you chose B:** You worked out 4 × 6 = 24 but did not count decimal places. Each number has 1, so the answer has 2. Go to M3.3.6.
   - **If you chose C:** That has 3 decimal places. 0.4 and 0.6 have 1 each, so the answer has 2 in all. Go to M3.3.6.

4. **2.4**

   Divide the ones: 7 ÷ 3 = 2, and 1 is left. 1 one and 2 tenths is 12 tenths. 12 ÷ 3 = 4 tenths. So 7.2 ÷ 3 = 2.4.

   - **If you chose A:** The digits are right, but you forgot the decimal point. 7.2 is about 7, and 7 ÷ 3 is about 2, so the answer is near 2. Go to M3.3.6.
   - **If you chose C:** The point is in the wrong place. 7.2 is about 7, and 7 ÷ 3 is about 2, not a quarter. Go to M3.3.6.

5. **11.89**

   4.1 is about 4 and 2.9 is about 3. 4 × 3 = 12, so the answer should be near 12. 41 × 29 = 1189, and 2 decimal places gives 11.89.

   - **If you chose A:** Estimate: 4 × 3 = 12. 118.9 is far too big. The point has moved too far to the right. Go to M3.3.6.
   - **If you chose B:** Estimate: 4 × 3 = 12. 1.189 is far too small. Count the decimal places: 1 + 1 = 2, so the answer has 2 places. Go to M3.3.6.

6. **14.4**

   36 × 4 = 144. 3.6 has 1 decimal place, so the answer is 14.4.

7. **0.28**

   7 × 4 = 28. There are 2 decimal places in all, so the answer is 0.28.

8. **2.4**

   Divide the ones: 9 ÷ 4 = 2, and 1 is left. 1 one and 6 tenths is 16 tenths. 16 ÷ 4 = 4 tenths. So the answer is 2.4.

9. **3.85**

   Write 6.3 as 6.30. Hundredths: 10 − 5 = 5. Tenths: 2 − 4 cannot be done, so borrow: 12 − 4 = 8. Ones: 5 − 2 = 3.

M3.3.7. Fractions and decimals

1. **0.6**

   $^3/_5$ means 3 ÷ 5. 30 tenths ÷ 5 = 6 tenths, so 3 ÷ 5 = 0.6.

   - **If you chose A:** You wrote the 3 and the 5 either side of a point. A fraction is a division. Work out 3 ÷ 5. Go to M3.3.7.
   - **If you chose C:** You divided 5 by 3, which gives 1.666... The top number is divided by the bottom number, so work out 3 ÷ 5. Go to M3.3.7.

2. **0.75**

   3 of 4 parts are shaded, so the fraction is $^3/_4$. 3 ÷ 4 = 0.75.

   - **If you chose A:** You wrote the 3 and the 4 either side of a point. Divide instead: 3 ÷ 4. Go to M3.3.7.
   - **If you chose B:** That is the part that is not shaded, $^1/_4$. Count the shaded parts, 3, and divide by the 4 parts. Go to M3.3.7.

3. **$^{3}/_{5}$**

   0.6 is 6 tenths, which is $^{6}/_{10}$. The HCF of 6 and 10 is 2, so $^{6}/_{10} = {}^{3}/_{5}$.

   - **If you chose B:** That is the right amount, but it is not in simplest form. 6 and 10 are both in the 2 times table, so divide both by 2. Go to M3.3.7.
   - **If you chose C:** You put the 6 on the bottom. 0.6 is 6 tenths, so the 6 goes on the top, over 10. Go to M3.3.7.

4. **$^{1}/_{8}$**

   0.125 is 125 thousandths, $^{125}/_{1000}$. The HCF of 125 and 1 000 is 125. 125 ÷ 125 = 1 and 1 000 ÷ 125 = 8.

   - **If you chose A:** That is the right amount, but it is not in simplest form. 125 and 1 000 share a factor of 125, so you can divide both by it. Go to M3.3.7.
   - **If you chose C:** You put the 125 on the bottom. 0.125 is 125 thousandths, so 125 goes on the top, over 1 000. Go to M3.3.7.

5. **$^{1}/_{3}$**

   1 ÷ 3 leaves 1 over every time, so the 3 repeats for ever: 0.333... The other two divisions end with nothing left over.

   - **If you chose A:** It stops at 0.375, as in this chapter. Nothing is left over, so it does not repeat. Go to M3.3.7.
   - **If you chose B:** Divide 1 by 4. You get 0.25, and nothing is left over. The decimal stops, so it does not repeat. Go to M3.3.7.

6. **0.625**

   5 ÷ 8: 50 ÷ 8 = 6 left 2, 20 ÷ 8 = 2 left 4, 40 ÷ 8 = 5. So 0.625.

7. **$^{9}/_{20}$**

   0.45 is 45 hundredths, $^{45}/_{100}$. The HCF of 45 and 100 is 5, so $^{45}/_{100} = {}^{9}/_{20}$.

8. **12**

   24 × 5 = 120. 2.4 has 1 decimal place, so the answer is 12.0, which is 12.

M3.3.8. Percentages

1. **$^{3}/_{5}$**

   60% is 60 out of 100, which is $^{60}/_{100}$. The HCF of 60 and 100 is 20. 60 ÷ 20 = 3 and 100 ÷ 20 = 5.

   - **If you chose B:** That is the right amount, but it is not in simplest form. 6 and 10 are both in the 2 times table, so divide both by 2. Go to M3.3.8.
   - **If you chose C:** 60% is 60 hundredths, not 6 hundredths. Write $^{60}/_{100}$ and simplify. Go to M3.3.8.

2. **0.08**

   8% is 8 out of 100. 8 ÷ 100 = 0.08.

   - **If you chose A:** 0.008 is 8 thousandths, which is 0.8%. Divide 8 by 100, not by 1 000. Go to M3.3.8.
   - **If you chose B:** 0.8 is 80%. 8% is 8 hundredths, which is 8 ÷ 100. Go to M3.3.8.

3. **15 kg**

   25% is a quarter. 60 ÷ 4 = 15, so 25% of 60 kg is 15 kg. The scale shows that 50% is 30 kg, and half of that is 15 kg.

   - **If you chose A:** 25 is the percentage, not the amount. 25% of the bar is a quarter of 60 kg. Go to M3.3.8.
   - **If you chose C:** You took 25 away from 60. A percentage of an amount is a part of it. 25% is a quarter, so divide 60 by 4. Go to M3.3.8.

4. **9**

   10% of 60 is 6. 5% is half of that, 3. 6 + 3 = 9.

   - **If you chose B:** You divided 60 by 15. 15% is 15 out of every 100. So build it: 10% of 60 is 6, and 5% is 3. Go to M3.3.8.
   - **If you chose C:** You added 15 to 60. A percentage of an amount is a part of that amount, so the answer is less than 60. Go to M3.3.8.

5. **30%**

   $^{12}/_{40}$ = 0.3, because 12 ÷ 40 = 0.3. 0.3 is 30 hundredths, which is 30%.

   - **If you chose A:** You divided 40 by 12, which gives 3.333... Divide the part by the whole: 12 ÷ 40 = 0.3, which is 30%. Go to M3.3.8.
   - **If you chose B:** 12% is 12 out of 100. Here the whole is 40, so write $^{12}/_{40}$ and change it into hundredths. Go to M3.3.8.

6. **18 kg**

   10% of 120 is 12. 5% is half of that, 6. 12 + 6 = 18.

7. **42%**

   $^{21}/_{50}$ = $^{42}/_{100}$, which is 42%.

8. **0.125**

   1 ÷ 8: 10 ÷ 8 = 1 left 2, 20 ÷ 8 = 2 left 4, 40 ÷ 8 = 5. So 0.125.

About this book

For learners who can show with a drawing that 2/4 is the same as 1/2, and who can add, take away, multiply and divide whole numbers. In many schools this fits children of about 11 to 13, and older learners who never got a firm hold of fractions or decimals can start here too. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it.

Written and drawn by attnom. Edition 1.0, 2026-11-12.

This book is free. You may read it, print it, copy it and share it with anyone.

The newest copy is at https://attnom.com/academy/fractions-decimals-and-percentages.